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Education in the War Economy*

By B. W. VENABLE

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FROM THE point of view of the army, the training in the schools and colleges falls into two general classifications. First, military training. Second, training of a military nature which may be of value to the military service.

Purely military training is carried on in many high schools for the benefit of students sufficiently mature to take it. It is called *ROTC* training. However, since it does not lead to a Reserve Officers Commission it might just as well be called *HSPT*, High School Pre-Induction Training.

The War Department is not in a position to expand the type of training at present. It requires close supervision by competent officers and the officers are not available for that type duty beyond our present commitments. I'm sure you will agree that the creation of an army second to none is the vitally important task before us today. Upon this task we must concentrate our entire efforts and energy, any diversions therefrom are not believed justified. Granting the necessity for adhering to the above policy you must look elsewhere for your service opportunities.

I know I can make the assumption that everyone desires to contribute to the national effort. It is perfectly natural and human to want to make that contribution tangible. Officers and soldiers in the army are eager to leave the humdrum administrative jobs for the more spectacular field service. We all want to dramatize our part in the effort by doing something out of the ordinary. We forget that the soldier in the field finds that war is marching and marching and halting and marching again with maddening monotony. He too would like something spectacular to do. Moments of combat, which are the final test of training, are brief and fleeting. General MacArthur and his brave command are getting more than their share of the spectacular—in fact so much that even the spectacular is becoming a bore, no doubt.

The high school is not faced with a depletion of male students to such an extent as is the college. You will not be required to dress up a lot of subjects in uniforms and parade them through catalogues as decoys to get students. Your algebra will not have to be starred as defense algebra, nor will your physics have to be called military physics. Yet you are desirous of doing your part and I can assure you that the War Department considers it most important that the high school student take advantage of every minute of education available to him now and in the future, eventually including a college course.

* Speech delivered before Section B of the Annual Convention of American Association of School Administrators, San Francisco, California on February 24, 1942.

If there must be a fanfare and military glamour present to attract, then let's have it; but the War Department hopes that the normal educational activities of the high schools will be interrupted as little as possible. It is still believed that a healthy training of mind and body is the best contribution the schools can make. The War Department cannot ignore the value of other subjects in Pre-Induction Training, but there are many subjects, dressed in khaki sometimes, that will have little or no application in the service, and we would like to offer a request that you not temporize with the situation. That you not stampede in all directions seeking some magic course because you will very likely find you have the essentials already in your curriculum.

In the complex pattern of modern warfare, the Army—somewhere in its intricate functioning—makes use of the results of nearly all human knowledge and research. Therefore, all educational processes may be considered part of a comprehensive military training. Actually, Pre-Induction Training starts in kindergarten and ends with induction, as almost every experience is a lesson.

The normal processes then of education need little or no adjustment to become valuable to our National effort. It is not so much the increase in number and variety of subjects available to the student as it is placing the proper emphasis on those courses already established that have a practical value, and teaching them in a practical way. More effort should be required of the student. A higher standard should be set. Lots of home work and midnight oil to the effect that A's, & B's will sprout where C's & D's have flourished heretofore.

The War Department believes that purely military training can best be done by the Army itself after the soldier is enrolled. The Army has excellent schools teaching all the special subjects, and its methods of teaching the most elementary duties are up-to-date. All men accepted into the Army have had some Pre-Induction Training and their progress in their new profession will reflect that training. Army training for the vast majority of our soldiers is simple and highly repetitious. The technique of most of the weapons is learned by rote. When we enter the realm of tactics, which is the essence of the art of war, we realize the need for imagination and ingenuity in the private soldier and especially in the leader.

In making an address before the Field Artillery School at Ft. Sill several years ago, a German Officer said in substance that the aim of the German Army was to teach each soldier to be a machine with a brain. His implication was that machine-like strengthened precision in carrying out orders is essential, but there must be sufficient mental capacity to understand the orders and carry them out intelligently.

Within an Infantry Division there are 11,500 Privates and Privates First Class. There are just under 4,000 Officers and Non-Commissioned Officers. They range from Corporals and Sergeants who command from eight to twelve men, to the Major General Commanding the Division. A few of the Privates are detailed to Administrative jobs as specialists, and a few of the Officers and Non-Commissioned Officers are detailed to Administrative jobs also, but for all practical purposes one out of four men in the Army is a leader or is expected to be a leader.

The colleges and universities of the country have been producing our leaders in both civil and military life for a number of years and have been doing a most satisfactory job. The need for leaders did not arise with the beginning of this war, nor will it end when this war ends. We must depend on our high schools and colleges to continue to produce more and better leaders, and it cannot be done satisfactorily if momentary enthusiasm blinds the students' view of the future.

It is difficult to suggest a program for teaching leadership. Leadership is a quality so fine that it is difficult to standardize it or even define it. We recognize a number of attributes that are common to successful

leaders. Character is the most important because character is the sum total of most of the intangibles that are essential to good leadership. Knowledge, judgment, sympathy, foresight, imagination, mental and physical coordination, strength of mind and body, self discipline, a stout heart, confidence, ambition and above all a will to fight and fight to destruction if necessary are some of the essentials of good leadership.

During the last World War 5% of the Army personnel were college men and 4% were high school graduates. Ninety-one percent of the Army never finished high school. Figures indicate that eleven percent of the present Army will come from the college world and 30 per cent will be high school graduates, leaving only 59 percent with less than a complete high school education. Those figures may vary considerably as mobilization continues, but they are none the less astounding. As the Army still needs its "Hewers of Wood," its "Drawers of Water," and as 25 percent represents the number of leaders, it appears that a large number of college and high school men will be in the private grade so we want to know what constitutes a good Private soldier. The qualities desired in a good Private, and in a good leader are the same. The selection from Private to leader may be based on something tangible or intangible, but there is no discrimination in the credit received for service freely given in any capacity.

The objective of Pre-Induction Training of a non-military nature during the last war, as stated in the report of the Provost Marshal General in 1920 was "To put the Selective Service men into camp, willing, loyal, intelligent, clean and sober, and thus fit him better for rapid progress in becoming a good soldier." We Americans dislike the word regimented because it implies restraint and subordination, but we know it means power. In preparing the state of mind of the prospective soldier to receive the shock of an entirely different environment and way of life it might be well to inject a little regimentation into institutions that haven't the ROTC. A soldier respects his superiors and shows them deference when they are together. Thus a student might be required to rise when an instructor enters a room, to be alert during classes, with feet on the floor and head up. This would require additional effort on the part of instructors, and a system of penalties and awards. This and many other requirements could instill that personal discipline that is so essential in a good soldier. That simple discipline akin to faith that helps a man subordinate his personal desires, comforts and pleasures to the will of a commander and the importance of the job at hand.

The most satisfactory way to sum up what is desired in a soldier or officer is for each one of you gentlemen to think of yourself as a Company or Battery Commander and that you are permitted to select your men. First of all you would want all soldiers and officers to be men of character who know the difference between right and wrong. Men you can trust with your life. Second, you would want men who have the will to fight to destruction if necessary, disciplined men who understand orders and obey them without question. Third, you would want men physically strong and clean, and then you want your men of skill, your expert rifleman, gunners and radio operators: squad and section leaders who understand minor tactics, a mess sergeant who knows how to plan meals, order and run a kitchen, trained cooks, competent supply sergeant, company clerk and company mechanic. Your Officers and First Sergeants must have all these attributes and skills and must be alert, careful, clever and aggressive if they are to succeed in modern combat.

We recognize character as almost synonymous with leadership. What has been said about teaching leadership applies also to teaching character and discipline. The Army needs men who are strong and rugged and well coordinated. The German

soldier today is probably the finest physical specimen since the days of Sparta, and the Jap, though smaller, is rugged and versatile. They are the soldiers we must train to whip. A complaint reached the War Department not long ago from one of our leading institutions to the effect that compulsory physical training was not a success because the students became bored. That institution needs something more fundamental than physical training, it needs physical discipline.

The application of courses the War Department suggests for improving the professional background of the soldier is in most cases elementary. It is suggested that fundamental subjects be taught rather than undertaking a cursory examination of some intricate specialty. A sound course in high school physics would certainly be of more value than a thin meaningless veneer in meteorology or military psychology. Progress to more advance college work would then be orderly and would supply the scientists and specialists required by the Armed Forces, and at the same time lay the foundation for a scientific career in civil life when the clouds of War have been dispelled.

Every man in the Army will at some time have to express himself orally or in writing. Thus the study of English is most important. The knowledge of certain foreign languages, Icelandic, Arabic, Japanese and Russian, is very important and anyone having knowledge of such language should make the fact known upon induction. Proficiency in foreign languages being very difficult without some background, the general study of those languages as an aid to the National effort is not suggested.

Next to English in importance is mathematics. Some specialist branches charged with designing airplanes, cannon, shells, etc., have use for the high mathematics. A useable knowledge of arithmetic will answer the needs of practically all the enlisted men and officers of infantry. Artillery, Engineers and Air Corps use trigonometry, navigation and plain surveying.

The natural sciences are subject to general use. Vocational and manual training have wide application in the Army. A soldier who cannot do things with his hands is helpless.

Most Army Officers of experience agree that the training of boy scouts is about the most complete all-around pre-induction training that can be given at that age level. It is a pleasure to receive men into the service or students into the ROTC who have had scout training because you know they have a clear cut sense of right and wrong, they live cleanly, they are independent yet know the value of cooperation They know the woods and the heavens; how to use a compass, how to cook and generally take care of themselves, and they know how to do things with their hands.

Physics is a valuable course. Its application is found in the Signal Corps in the theory of electricity as applied to the telephone, telegraph, radio and radio interception. Other branches use the same means of communication but there interest generally stops at operation and maintenance.

In the Air Corps the application of physics goes beyond applied electricity to include heat, light, sound, meteorology, and aerodynamics.

The internal combustion engine is the prime mover of the Army today. Every branch of the service has its drivers and mechanics. Operation and light maintenance is all that is required of the vast majority of drivers, but a knowledge, gained from the study of physics, of how the ignition, fuel and cooling systems cooperate will prevent abuse and add to the life and usefulness of the machine.

We must not forget that there are other agencies of the Government besides the Armed Forces that are prosecuting this War. The study of chemistry has only slight application in the Army. True we have chemical warfare, but the chemicals

used are not prepared by soldiers and the methods of dissemination are similar to the methods used in throwing mortar and artillery shell at a hostile force. To use a gas mask or recognize the odor of a gas, and to understand something of decontamination, does not require a course in chemistry. Training in chemistry then must be generally recognized as training for essential industry.

Representatives of the Council on education have recently been in my office and have visited throughout the War Department, Navy Department and the Air Corps. They have called on all Branch Chiefs and received the best opinion available as to what types of Pre-induction Training are pertinent. A report of the result of this investigation will undoubtedly be available in the near future. Copies may be obtained from the American Council on Education, 744 Jackson Place, Washington, D. C.

It requires no prophet to predict that the result will be a call for more emphasis on the fundamentals. "There never have been and there probably never will be enough men of leadership, character and stout hearts."

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THE MATHEMATICS TEACHER

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The Navy and the Schools*

By PAUL C. SMITH

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WE ARE engaged in a Total War. Total war, as conceived by our enemies, means that *all* the resources of their nations have been mobilized for a single purpose—war. And, as we must realize, it is therefore necessary to mobilize *all* the resources of this nation of ours in order to achieve final victory over our totalitarian enemies. Since dictators have been quick to recognize that education is one of the greatest resources of any nation—and certainly its most powerful directive force—they have ruthlessly commandeered the schools and those who teach in them. In Germany, in Japan, and in Italy, the educational establishments have worked for years in closest collaboration with the military forces. Educational psychologists have been made an actual part of the enemies' armed forces and military men have been placed in administrative positions in their schools. The State tells the schools in no uncertain terms what their part in the Total War effort will be, and sees to it—with the aid of the lash or castor oil or the firing squad if necessary—that its instructions for the education of totalitarian youth are carried out.

That is the Nazi and Facist and Nipponese idea of collaboration. That is the totalitarian way of doing things. We, the people of the United States, do not and will not employ such methods.

Nevertheless, there is a real and vital need in this hour or crisis for close cooperation between our educational institutions and our military forces. While we have been writing theses on the Influences of the City of Bath on 18th Century English Literature, Nazi professors have written scholarly tomes on just how much specific torture is necessary before a terror psychology is induced in a given area. While we have prided ourselves on giving our students a free hand in their exploration for knowledge, Italian schools have integrated every school subject to the end of military efficiency. While we have taught peace, Japan has taught war.

We Americans hate this conception—I should more accurately say this *prostitution*—of education. It is intolerable to our way of life. Yet when the first bomb fell out *there*, we came up against the cold fact that our way of life is in danger. American boys at this very moment are fighting the fierce robots that have poured from the totalitarian educational mills. America is awakening to the fact that its schools, founded in the principle of democracy, can play an all-important part in preserving that democracy. And there is not a moment to lose.

But, you say, isn't it already too late? After all, education takes time. Years will pass before the students in our classrooms will be fighting for their country. Yes, but probably years will pass before the issues of this struggle are resolved in peace. There is only one way to win this war, and that is to fight until the enemy gives up. Therefore, no one can say we will fight two years or five years or ten years. We are simply going to fight until we win. That must be the basis for our planning. Hundreds of thousands of boys now in high school will be manning America's ships and tanks and planes within the next two or three years.

More than any other war in history, this is a war of youths. In Caesar's time, when broadsword clashed against shield on the field of battle, the warrior was usually a mature man whose muscles had been hardened by years of tilling the fields. But as warfare has become more and more mechanized, the fighting man has become

* Speech delivered before Section B of the Annual Convention of American Association of School Administrators, San Francisco, California, on February 24, 1942.

more and more youthful. The sailor most useful today at the controls of a gun turret is the one who learned instinctively to calculate the speed of traffic as soon as he could walk, and who spent his early boyhood taking apart and putting together again the innards of a jalopy.

Machines of war, like machines of peace, have long since become faster than all the keenest eye and the quickest muscle.

That is why you, the educators of the country, who are most experienced in molding youth, stand at the springs from which our war efforts flow. The young men under your educational guidance are in their most formative period.

Let me hasten to assure you right now, as you will see for yourselves in a few moments, that the Navy does not ask you to change your curriculum or revolutionize your schedule to turn out fighting machines. In fact, they do not even ask that you give any kind of military training. The kind of instruction that is useful to the armed forces is the very kind of instruction that will be of most use to the citizen in the peacetime world that is to follow. It is striking evidence of the impeccable rightness of our fight that the subjects which help America wage the war are the subjects which will help build the peace after the holocaust is over.

How, then, can you help?

First, and foremost, you can help the Navy immeasurably by the simple expediency of stressing those of your regular subjects in which the Navy has found its present recruits most lacking. Mathematics is the most important of these, followed closely by sciences and manual subjects.

It is easy to see why such courses are absolutely essential to basic Naval training. In contrast to the Army, the basic fighting unit of the Navy is not the individual soldier but the ship. Therefore, while a soldier is able to throw himself into battle without outside help, and while only five men are required to throw a tank into battle, it requires the highly specialized and perfectly integrated efforts of no less than 1500 men to throw a battleship into battle. That means that every man on board must be a highly trained technical expert in his own job. To be able to absorb the training in the short time allowed in wartime, he must be well grounded in mathematics, the better to work out problems of mechanics or marksmanship—in science the better to understand electric power and pressures and friction and weights and properties of explosives—and in manual subjects, the better to diagnose a sputtering motor, or grind a piston.

So the Navy asks that you encourage every male student under your supervision, who has the slightest aptitude for such subsects, to take an absolute minimum of two and one-half years of mathematics. Teach him algebra through quadratics, plane and solid geometry, and trigonometry. Ground him well in chemistry and physics. Give him practice in machine shop work, foundry work, metalworking, woodworking, and internal combustion engineering.

The most important group, because it is the most basic, is mathematics. I hope I won't offend any of you distinguished educators when I say that I personally loathe mathematics. But, as a naval officer, I recognize how essential they are.

Put yourselves, for a moment, in the shoes of a young commanding officer on the bridge of a destroyer in enemy waters. From the deck below, the anti-aircraft lookout, sweeping the horizon with his binoculars, bawls out, "Stukas off the port bow!" A moment later, the faint hum of their motors crashes to a roar as their three-hundred-mile-an-hour speed brings them down on you and your ship. As you peer skyward, facing death, the leader peels off, points his nose almost straight down, and hurtles toward your ship. As your anti-aircraft guns start to bark, you begin making lightning calculations. The height of the plane—about 3000 feet. Its speed, mounting in acceleration. The angle of its dive and the spot at which it will release its bomb. The speed of the ship and

the time it will take to respond to a frantic, "Right full rudder." On the result of these mental gymnastics, necessarily performed in a split-second's time, will depend 150* lives, a ship of the U. S. Navy, and perhaps the success of an entire Naval engagement.

Yes, mathematics—and science and manual subjects as well—are acutely necessary in the Navy. But they will be equally necessary in the peace that follows. The forces of destruction that have been unleashed upon the world have been planned by the most scientific minds in the world. The minds which rebuild after this destruction must be equally scientific and even more ingenious. It will take educated and trained men to heal civilization's wounds.

Wherever it is feasible, you can help the Navy by instituting certain temporary elective courses in addition to the regular curriculum. A short, practical course in instruction in the International Morse Code is among those needed. Other valuable subjects would be elementary principles of radio communication, principles of electricity and of navigation, and naval history.

You can help a great deal by making it clear to the students how these subjects will help them serve their country when their times comes. You need make no secret of the fact that a knowledge of these subjects is apt to speed up advancement in the Service. You might do this by a short talk in assemblies at the beginning of each semester, by personal talks with those undecided on their schedules, by bulletins or even posters on your bulletin boards, or by mimeographed general announcements. You probably have all seen the folder on this subject prepared by the Navy. It enumerates the specific educational prerequisites for each specialized job in the Navy, together with an outline of the training the Navy itself offers. See that every male student sees one of these folders. If any additional are desired, they may be obtained by writing the Training Division, Bureau of Navigation, Navy Department, Washington, D. C.

You can help the Navy immensely by laying a great deal more stress on physical education. Again let me remind you that the nations we are fighting have long glorified physical development as the highest attainment of youth. But their methods of insuring this physical development have, for the most part, been physically tyrannical. Mass calisthenics, state-sponsored health camps, and compulsory hard labor prior to military training have been among the devices used by the dictators.

We in America instinctively abhor such regimentation. But our need for strength and vitality is no less imperative. The high percentage of Selective Service rejections due to physical disabilities has been shocking. And the Navy found that so many volunteers were underweight, sunken-chested, weak-eyed, soft-boned, flabby-muscled and so on, ad infinitum, that it has been forced to set up an intensive physical fitness program within its own ranks. The results of a few weeks of directed exercise, healthy habits, voluntary sports have been almost unbelievable.

In American, the place for physical education should not be in compulsory camps, as in Germany, nor within the armed services themselves. The place to build bodies, as well as minds, is in the high schools and junior high schools, while those bodies are in a formative period. American schools have probably the finest facilities for physical education of any of the educational establishments on the face of the earth. And in the light of such facts, there is no reason why the graduates of those schools should not be the healthiest and strongest in the world. They *must*, if we are to win this war. So through these means—talks in assemblies, personal interviews, posters and bulletin board announcements, and mimeographed bulletins—carry on a continuous campaign to arouse enthusiasm in sports and physical education. Show every student how personal prowess will not only help his coun-

* (This figure appears in original. Subject to check.)

try crush its enemies but will mean a longer, happier, and healthier life for him in the peace time that follows.

Now, as you probably know, the Navy depends entirely on volunteers to man its ships and planes. The Navy predicates this policy on the sound belief that in the cramped quarters below decks in a warship where every person must function with perfect efficiency for the safety of all, there is no room for anyone who doesn't *want* to be there. On the other hand, however, this policy creates the necessity of keeping the young men of America constantly interested in the Navy and constantly aware of its high purposes in the interests of the Nation.

Once again, because the high school and junior high school boy is most receptive to ideas—because he is at his turning point—it is imperative to the future operation of our Navy that the schools of the nation help him to know as much about it as possible. One way you can help the Navy, then, is to suggest organization of school assemblies around Navy themes. Meetings might feature a colorful talk on a subject such as *The Navy as a Career*, *Stories of Naval Heroes*, *The Life of a Bluejacket*, or *Navy Ships and Planes*. The Office of Public Relations of the Navy Department in Washington is prepared to supply you with material on these and similar subjects. The speakers might be either local Navy officials or school authorities.

Also, you might devote one of the school bulletin boards to a Navy recruiting poster which your local recruiting station would be glad to supply you. This is not, mind you, with any idea of gaining a single immediate enlistment, but simply for the purpose of maintaining a constant *interest* in the Navy. Your school newspapers and school annuals also constitute splendid media for the dissemination of Navy material. Your students are undoubtedly interested in articles on the Navy. Why not plan a series of them to run in your school publications.

You can help sell the parents of your students on the Navy, too. The Navy insists that every volunteer under 21 years of age obtain the written consent of his parents before he is accepted. So selling the enlistee is often the easier half of the Navy's selling job. Because of the sincere confidence imposed in the teachers and administrators of this country's schools by the families of the students who attend them, P.T.A. meetings afford an ideal means of reaching the parents with information about the Navy. We suggest that you hold at least one meeting each year with the express purpose of furnishing information about the Navy. In these times, anxious parents are hungry for the information. They will thank you for helping them obtain it. And the Navy will thank you, too.

There is one specialized problem of student guidance that deserves special attention for a moment. I know that every one of you is frequently called upon by active-minded, red-blooded patriotic young men to help decide first whether they shall go on to college after high school or join an armed service, and secondly, what service they shall choose. As you deeply appreciate, this is a task of guidance which far overshadows in importance any of the others you are called upon to perform. On your judgment depends, to some degree, the safety of the nation as well as the future of the individual.

When confronted with this problem, consider it from two viewpoints. First, would the student be good college material in normal times, or would he be happier in a job? Secondly, would he, in your opinion, be of more value to a fighting service as a leader or a follower? Often, though not necessarily, the answers from both viewpoints will coincide. If the decision is to continue to college, point out that two years of college training makes him eligible for training as a Naval officer. And if the decision is to the contrary, you might advise him to enlist immediately upon graduation from high school and give him the assurance that the Navy will help

prepare him, if qualified, for a trade useful in peacetime. You are a kind of gigantic pre-selective service board, and your responsibility is immense.

In conclusion, you can help the Navy by inculcating in today's young people the principles of Democracy and self-discipline and humanity which have been so long neglected. This is no empty oratory. To sense the importance of such teachings in a democracy at war, ask yourself, "What do men fight for?" There are a hundred answers. They fight for conquest, for plunder, for their homes, for the fun of fighting, and no end of other things. But we in America are fighting for an *idea*—freedom, human dignity, liberty, democracy—whichever you may choose to call that idea that is the ideal behind American life. And ideas are your province. They are the tools with which you work. It is *your* job to impress indelibly on the youth of America the absolute rightness of our way of life—of democracy and freedom as opposed to tyranny and degradation. Impress on them the overwhelming necessity of defending that way of life with their blood and with their lives.

I've tried to impress upon you the importance of technologists in modern warfare. But considered simply as a technologist, there might be a little difference between a Nazi technologist and an American technologist. The difference in the outcome of the war, then, will resolve itself into the difference between the basic convictions—the ideals—over which the two technologists have been warring. Recognizing this, the Axis countries have stuffed their subjects' skulls with totalitarian ideology. Certainly, then the least we can do is imbue our own youth with a love of the democracy in which they have been raised. The least we can do is to seek to guide them to an unquenchable faith in the rightness of freedom and individual human dignity.

If our teachers and educators can do this, they will have played a vital role in the winning of this war and done as much as any other single human agency to assure that the coming victory shall bring more than bare survival and temporary security. They will also have contributed to the even bigger and more complex victory—the winning of the ultimate peace.

THE ALGEBRA OF OMAR KHAYYAM

By DAOUD S. KASIR, Ph.D.

The author of the *Rubaiyat* was also an astronomer and mathematician. This work presents for the first time in English a translation of his algebra. In the introduction, Mr. Kasir traces the influence of earlier Greek and Arab achievements in mathematics upon the algebra of Omar Khayyam and in turn the influence of his work upon mathematics in Persia in the Middle Ages. The translation is divided into chapters, and each section is followed by bibliographical and explanatory notes.

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Mathematics in the National Emergency*

By E. R. Hedrick
University of California at Los Angeles, California

I HAVE spoken frequently to your organization and I am happy to be with you again today. I hope that I shall not weary you with too much repetition of my own past statements. At the least, you know where I stand. When he reported to his wife on a sermon entitled "Sin," the late President Coolidge is said to have told her only, "He was against it." In the same admirably brief manner, as for the teaching of mathematics in our schools, you may safely report that *I favor it.*

In the past ten or twenty years, as I have remarked elsewhere, the teaching of mathematics in schools has *progressively* degenerated. Against that tendency, I have spoken repeatedly in the past, to you, to engineers, to superintendents and principals of schools, to educators, to the public, in short, to all whom I could induce to listen to me. Nor have I been alone: every thoughtful school leader not bound by loyalties to those who form a sort of dynasty of education have seen the mistake and have warned as I have warned against it.

Now we are at war. The nation seeks and must have an enormous number of trained persons—mechanics, air pilots, navigators, officers for artillery for sound ranging, for the Army, the Navy, the Marine Corps. You have seen in print and have heard over the radio the specifications repeated again and again and again: "mathematics," "mathematics and physics," "mathematics, physics and chemistry." A little later I shall read one document to you. However, it is obvious in advance that whenever quantities and relations between quantities are to be dealt with, training in mathematics is involved, since mathematics is precisely the study of quantitative relations.

Every officer, every mechanic, every navigator must deal with quantities and their relations. He must work quickly and accurately. Mistakes are exceedingly dangerous. In ordinary life, as I have said elsewhere, quantities occur in the lives of all active persons. It is folly to say that they do not. It is folly not to prepare people for life. Such folly, to our sorrow, has penetrated to very high places in ourschools.

Now we reap the harvest of such folly. Faced with terrible emergency we cannot find enough young people who can handle quantities safely and accurately. Every service is scouring the country for such people, for everything from work in airplane factories and shipyards to the skilled work required for air pilots, navigators, gunners, officers of every armed force. We lack not only munitions and mechanized equipment; we lack enough young people to make and to manage them skillfully and correctly. This is a disaster at such a time, comparable to that of Pearl Harbor in its effects upon the nation. It is the "Pearl Harbor" of our education. Two weeks ago at Bakersfield, Mrs. May of the *ESMDT* told a convention of principals and teachers in junior colleges that the Government has now determined to train thousands of young *women* who have two years of college education to replace men in airplane factories, shipyards, etc. The announcement was received with great enthusiasm. Mrs. May then went on to say that those selected should have studied mathematics and possibly also physics or chemistry. There was a sigh that was almost a groan when the audience heard that. Too many women have been told that a woman does not need algebra, nor even arithmetic.

* Read before the National Council of Teachers of Mathematics at San Francisco, February 20, 1942.

Have I exaggerated? Are the young people graduated from our secondary schools under the "*laissez-faire*" policies of recent years not capable of at least simple mathematics? To determine this, many colleges have instituted elementary tests for freshmen; such tests have been made simpler, as experience dictated that any serious test was quite useless. I shall quote a few sample questions presently.

One very extensive test is conducted annually, by a joint agreement, by 29 colleges in the State of Kansas, and statistical reports are published each year in the *Bulletin* of the Kansas Association of Teachers of Mathematics. Since a large number of students are involved and since the statistics are available, I have quoted from them on occasion. Last Spring I quoted extensively from the 1940 report in a paper that I read before the S.P.E.E. at their meeting in Ann Arbor. That paper is published in the September 1941 issue of the *Journal of Engineering Education*.

While I shall not repeat that here, I do have now the report for 1941 published in the October 1941 issue of that same Kansas *Bulletin*, and I would like to give a very few items from it. In passing, I wish to record my own profound sorrow, which many of you share with me, that Professor U. G. Mitchell, one of the leaders in the Kansas movement, died early in January. He had written me only a few days before his death, to comment on this report.

The tests given are in two parts: I. Arithmetic, II. Elementary Algebra. Each part consists of twenty questions, so simple that only fifteen minutes is allowed for each of the two parts. These tests were taken in 1941 by 4317 students in the 29 Kansas colleges. You can secure the full report from the Editor, Miss Ina E. Holroyd, Kansas State College, Manhattan, Kansas.

On the twenty questions on arithmetic, the median score was nine correct out of twenty. Perhaps you think the test was too hard. The first question was to square the number 70. Only 1096 of these college students failed to do so. That was pretty good It was one out of eight of the twenty that at least half of the students could do. The other twelve questions were done correctly by less than half the students. What are some of the "hard" questions? Here is one that only 308 out of the 4317 students could do: "If the area of a circle is 7.8 square inches, what is the area of a circle whose diameter is ten times that of the first circle?" Just about one student in fourteen could answer that. Exactly one student in four could write 1127 millionths as a decimal. One that amuses me vastly is the question: "Which is the larger 7/15 or 11/23?" Of the 4317 college students, only 1284 did that correctly. Some 1500 did not even try. But 1488 tried it and got the wrong answer. The joke is the same that I pointed out for a similar question on the 1940 report: the number of correct answers would have been greater if they had simply *guessed*, which would have resulted at least in a 50–50 score.

The part on Algebra is still worse, however. The median number of correct answers is *three* out of *twenty*. Of those who had algebra in high school, 438 out of 4317 failed to answer even one question correctly.

No matter! Doubtless these same young people can discourse learnedly upon the New Social Order! Or can they?

You will be saying: "So this is Kansas!" No; I repeat: It is Pearl Harbor. But the majority here are neither from Kansas nor Pearl Harbor, but from the Golden State of California. You will be smiling: "It can't happen here!" "Certainly," you will be saying, "nothing of the sort could happen at the University of California, for only the better half of the high school graduates can enter that University." Dean Hildebrand of the College of Letters and Science at Berkeley is also a Professor of Chemistry. He has allowed me to see a manuscript which I hope will be published soon;* it is entitled "*War and the Decimal*

* Published in School and Society, May 16, 1942.

Point"; and it begins with this original and delectable limerick:

While working at school,
Jill invented a rule
For coping with decimal fractions:
All those darn little dots
She *ignores;* it helps lots
With divisions, and sums, and subtractions.

He, too, is disturbed about mathematics, for quantities and relations between quantities occur also in chemistry. He tells here of a test on arithmetic given to one thousand freshman students in chemistry at Berkeley. One day I hope you will see all of it and his comments. I need tell you only a few items. Thus of the superlative students who are not only admitted at Berkeley but also are in chemistry, over 14% could not divide 0.175 by 25. Of the graduates from one certain secondary school, 31% could not do that correctly. Another question was to find the percentage of substance B in a compound consisting of 3 parts of substance A and 5 parts of substance B. Of the graduates from a certain secondary school, sixty per cent failed on this question. A third question was to find the cubical contents of a cylinder 6 cm. in diameter and 10 cm. high. Approximately half of all the students failed on that.

Since some persons feel that students absorb about the same amount from any procedure, it is important to observe that students from one large high school received more than *twice* as many high grades in this test as did those from another large school, proportionately. The nature of the training does show up. Similar results in Kansas are mentioned in my S.P.E.E. paper.

Such then, even at Berkeley, even among students who elect chemistry, are the results of the tampering with mathematical training conducted by those in charge of education in this country. Who are we to criticize the admirals and generals of Pearl Harbor. Were *they* more careless of national needs?

Shall we be reconciled by the thought that, while ignorant in arithmetic, these young people know all about the Wave of the Future and the Postwar World. Maybe they do.

I have already mentioned the repeated public statements regarding the need for mathematical training for the thousands of young persons of both sexes now imperatively needed. Let me read you an explicit statement from Admiral Nimitz, who is now in command of our fleet in the Pacific. In a letter dated November 12, 1941, while he was still head of the Bureau of Navigation, U. S. Navy, he said to Professor A. H. White, President of the Society for the Promotion of Engineering Education:

> Your letter of October 29 is rather difficult to answer with specific information as to the relative merits of progressive education and the more conservative methods. To the best of my knowledge no accurate record has been kept correlating the type of secondary schools or preparatory schools of each candidate for officer training with the record of the said candidate.
>
> I am, however, in a position to give you some information about established deficiencies which we have regretfully been compelled to recognize and you may draw such conclusions as you like from the facts which follow.
>
> A carefully prepared selective examination was given to 4200 entering freshmen at 27 of the leading universities and colleges of the United States. Sixty-eight per cent of the men taking this examination were unable to pass the arithmetical reasoning test. Sixty-two per cent failed the whole test, which included also arithmetical combinations, vocabulary, and spatial relations. The majority of failures were not merely borderline, but were far below passing grade. Of the 4200 entering freshmen who wished to enter the Naval Reserve Officers' Training Corps, only ten per cent had already taken elementary trigonometry in the high schools from which they had graduated. Only twenty-three per cent of the 4200 had taken more than one and a half years of mathematics in high school.
>
> The experience which the Navy has had in attempting to teach navigation in the Naval Reserve Officers' Training Corps Units and in the Naval Reserve Midshipmen Training Program (V-7) indicates that seventy-five per cent of the failures in the study of navigation must be attributed to the lack of adequate knowledge of mathematics. Since mathematics is also necessary in fire control and in many other vital branches of the naval officer's profession, it can readily be understood that a candidate for training for a commission in the Naval Reserve cannot be regarded as good material unless he has taken sufficient mathematics.

The Navy depends for its efficiency upon trained men. The men are trained at schools conducted for this purpose and the admission of men to these schools is based upon the meeting of certain carefully established requirements. However, in order to enroll the necessary number of men in the training schools, it was found necessary at one of the training stations to lower the standards in 50% of the admissions. This necessity is attributed to a deficiency in the early educations of the men involved. The requirements had to be lowered in the field of arithmetical attainment. Relative to the results obtained in the General Classification Test, the lowest category of achievement was in arithmetic.

A study has been made of the grades received in the examinations of candidates for enlistment in the Navy, classified geographically according to the location of the recruiting station through which the candidates applied for enlistment. It is to be noted that the proficiency in arithmetic in the eastern part of the country was strikingly greater than that of the middle west and west. The lowest average mark east of the Mississippi was equal to the highest average mark west of the Mississippi. The three highest average attainments in arithmetic were achieved by the recruiting stations in Troy, Brooklyn, and Buffalo—all in New York State.

I trust that this information will be of assistance to you.

Such reactions are finding public expression in many unexpected places. The issue of the magazine *Time* for January 5, 1942 contains a striking article on page 39. In the *Bulletin of Consumers' Research* for February, 1942, the editor has a long and powerful editorial, from which I would quote a single paragraph:

Every sort of genuine skill and competence will now be at a premium in a world dominated by machines, and, for the moment, by the machines and munitions of war. In that kind of world, that country is most ready and best qualified to meet emergencies which has graduated the largest number of young people from technical and business-training schools and colleges, equipped with the hard-won knowledge of mathematics, physics, chemistry, and the other applied sciences and technologies applicable to industrial and military uses. For most young people, the only way in which this knowledge and skill can be acquired will be from teachers who are versed in the subject matter of those arts and sciences, who have worked at them intensively in the shop and laboratory or study—or who have, if possible, written books and papers on them, made hobbies of them for their spare time.

I have heard it said, and I myself have thought, that the mistakes and oversights made at Pearl Harbor may be of real advantage in the end, because they have awakened us all to realization of the need for carefulness, alertness, and competence. In the same sense, we might hope that the demonstrated weakness in mathematics and associated sciences would result in an awakening in educational circles, and a return to more reasonable attention to these subjects in the schools. The educational journals, however, remain as blind to these needs as ever. There are few if any signs of recognition of the mistakes of the past and of the needs of the present. I have been obliged to quote from such journals as *Time* and *Consumers' Research*, rather than from journals on education.* Is it possible that "the old school tie" is as potent in maintaining wilful neglect of needs and in maintaining incompetent leadership in education in America as it has been, notoriously, in British politics and in the British armed forces?

Indeed, we see just now a resurgence of certain contentions. In an article that appeared in the *Los Angeles Times* on February 11, 1942, and doubtless in many other papers, the Commission on the Relation of School and College quotes from the report of Dr. W. M. Aiken, a statement that students who took little or no mathematics in secondary schools of the so-called experimental type made as good or higher grades in college mathematics and other college courses than did students from traditional high schools. What is not said is that students who had no mathematics in the secondary schools took only very elementary courses—courses that we in the colleges have been forced to conduct for them in topics that are normally given in high schools. In such childish courses,

* Since this manuscript was typed, I have seen the issue of *School and Society* for February 14, 1942, in which there are some articles that do recognize the situation, on pages 178–9. These include, notably, a copy of Admiral Nimitz' letter which I have mentioned, and which also appeared in the February 1942 MATHEMATICS TEACHER.

indeed, these students do as well or better than do properly prepared students in courses that are really of college grade. The spirit of the report is given by one sentence: "This slap at the three R's is based on an intensive comparison of progressive and traditional education." I believe that it is high time that educators cease to "slap at the three R's."

Nevertheless, I expect that at this very meeting of the N.E.A. with which this meeting is associated, speeches will abound in which more "slaps at the three R's" will be in evidence. Such exhibitions of prejudice and lack of comprehension of the needs of students and of society are by now well-nigh intolerable.

In an article in *School and Society* for February 7, 1942, Professor Briggs of the Teachers College at Columbia University has an article "*Cassandra Speaks.*" In it, he specifically warns against the teaching of arithmetic, which he evidently regards as evil. He says:

"*Education will make no significant contribution to democracy if it continues to teach the traditional curriculum, even if it labels the courses 'Arithmetic in the Present Emergency,' 'Geography in War Times' or 'Penmanship in a Period of Crisis.*"

"*The primary need, as I have been emphasizing for the past two years, is for us to clarify the understanding, first by ourselves and then by others, of the adult public as well as our pupils, of the meanings of democracy.*"

He then proceeds to outline, as the predominant need of all school training, discussions of the meaning of democracy and other kindred topics.

I believe with Professor Briggs that the schools should give very considerable attention to social sciences, and to such questions as the meaning of democracy. Where we differ is in the amount of time to be devoted to those subjects. Surely not all of every hour of every year ought to be filled up with these subjects. Surely the schools may spend a part of their time on the hated "three R's" without undue neglect of democracy. Surely we who seek some training in such vital things as reading, writing and arithmetic cannot be accused—on that account—of an attempt to suppress the teaching of quite adequate courses in social science. I, for one, though I demand that some adequate attention be paid to the three R's (so gleefully "slapped" by the Commission quoted above), still demand also adequate attention to the social sciences. I deny that all the time need be devoted to any one field of study. I deny that adequate attention to one precludes adequate attention to the other in our public schools.

But I carry coals to Newcastle if I continue to advocate to you—teachers of mathematics—that mathematics should be taught more adequately. We, too, may suffer from the dry rot of complacent snobbery if we only congratulate each other, if we—so to say—adopt our own "school tie" to proclaim our own assumed superiority. Rather we must ourselves consider the real needs of the whole public, and we must assess our part in it with due modesty and with recognition of our own past mistakes. For we, too, have sinned.

When mathematics was a general school requirement, not only in the graded schools but also in secondary schools, and even in the colleges and universities, we sinned in the face of every decent precept. We taught what we pleased, with too little thought of the public good. We earned the reputation which mathematics itself does not deserve, namely, that our work was "*disciplinary*" rather than for any usefulness. We taught square and cube root, we taught the Euclidean process for highest common divisor, we plagued students with such atrocities as fractions with six stories, and with adding fractions whose denominators were unconscionably huge. Little wonder that we earned the reputation of being pedantic taskmasters of a meaningless technique.

In higher courses, we sought to pile up tremendous—and often perfectly useless—machinery of our own delight. Entrance examinations vied with textbooks to see which could outstrip the other in such

trivialities as complicated factoring and solution of simultaneous quadratics. Nor do I condemn alone the elementary schools. College courses abounded, and they still abound, in perfectly absurd complexities of little value even to pure mathematics. Thus I myself studied spherical trigonometry from a book of over five hundred pages, whereas all of spherical trigonometry that is necessary even for navigation and for geodetic surveying can be stated on twenty pages. Nor were the remaining four hundred eighty pages of my book ever of even the slightest use to me in my later studies of mathematics itself. Shall I give further instances? I personally not only knew, but learned by heart all of the so-called reduction formulas for integrals, both algebraic and trigonometric. If the enormity of such teaching as that does not immediately impress you, will you not look up those formulas? To what end was I taught them and made to learn them by heart? I can cite literally hundreds of other instances of perfectly foolhardy things to which I myself and many other students of mathematics have been subjected.

At long last, then, I come to what my title may have suggested. What shall we teach in our own classes, particularly in those with vocational aims? I have believed it necessary to present all that precedes in order to treat this subject. I shall now be reasonably brief. I shall not attempt an outline of topics, nor would you care to hear it if I did.

What I am going to try to do is to outline policies. First of all, we must face frankly the appalling lack of knowledge of the most elementary arithmetic. It will do no good to rave against those who have caused this, when students face us. Such argument ought to be done in public rather than in classes. We need to resolve to teach the most elementary things: how to add $\frac{1}{2}$ and $\frac{2}{3}$, how to write in decimal notation, how to divide one fraction by another, . . . all of the simple first principles. Certainly in any course for vocational purposes, such elementary things are vitally needed. We should not be too proud or too snobbish to teach even these simplest things to those who (probably through no fault of their own) do not know them.

Next, we should forget all about "disciplining" our students. I have no objection to hard work, nor to making others work. What I do abhor is "*made*" work. There is enough hard work in mathematics to satisfy the most exacting taskmaster in learning things that are worth while in themselves and that are peremptorily needed in the work of the world—in vocations, if you please.

These principles appear to me to apply in every mathematical course. In algebra, for example, we should make sure of extremely elementary things: the Kansas test in 1941 contained four simple equations out of a total of twenty questions. Only one of these four simple equations was solved by a majority; even that one found 1511 students unable to solve it; the most difficult (still simple) equation was solved by only 110 out of 4317 college students. In factoring, the simple example x^2-4 was correctly factored by less than half of these 4000 college students. Why talk of the intricacies of the so-called "factor theorem" when such a situation exists? Let us make sure of very simple things. Only 94 of these 4000 could plot the graph of $2x=y+4$. Less than half could multiply $-2x$ by $3-x$. In the face of such a situation we can forgo simultaneous quadratics and even such things as the relation between coefficients and roots of algebraic equations.

In geometry and in trigonometry the story is the same. Do not bother about inscribing pentagons when the students cannot find their way around a right triangle. Make sure of the very simplest properties of triangles and of circles. Make sure of the most elementary mensuration formulas. Do not forget that half of the chemistry students in Berkeley could not find the volume of a simple right cylinder. In the face of such things, do not bother

much about the so-called incommensurable cases; in passing I may remark that the "proofs" for those incommensurable cases in many school texts are actually fallacious, anyway.

In trigonometry, make sure of right triangles first. Do not go to extremes in the so-called identity problems unless the students can solve triangles. Do not bother about intricate cases of use of the law of tangents until the laws of sines and of cosines are very clear. Pay more attention to the graphs of the simple functions than to memorization of the half-angle formulas.

In still more advanced courses, similar remarks seem to me to be pertinent. I have outlined some of these in my *S.P.E.E.* paper mentioned above. These I shall therefore omit here.

If we can adhere to the principles I have mentioned, we shall teach for excellent vocational results; but what is perhaps more surprising, we shall gain also in true mathematical insight. If we forget about "disciplining" our students and turn our attention to the teaching of things that are surely worth while, instead of bothering with "made" work, we shall benefit society, we shall benefit the students; and what is more remarkable, they will work harder than they ever worked on "made" work.

In general, then, I propose that you campaign for revival of real teaching of worth while subjects in our schools. By the same token, I exhort you to make very sure that your own teaching of mathematics consists solely of things that are worth while. By doing so, you will do your duty, and you will restore mathematics to its proper place in public esteem.

I propose that we arouse the country against the Pearl Harbor of our schools, and that we refuse to bend the knee to the wearers of the old-school-tie of the educational hierarchy. By the same token, I adjure you to absolve your own company from complacent snobbery of the very same sort, and to resolve that mathematical teaching shall be for the best good of society.

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Mathematics in Present Day Industry*

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WE ARE accustomed to thinking of mathematics as a formal theoretical science. As such, it is a most necessary tool for the physicist, the astronomer and other scientists engaged in profound scientific research.

When an attempt is made to connect the term "mathematics" with the everyday duties and jobs common in modern industry there seems to be a differentiation wherein mathematics becomes something on a higher level than arithmetic. In other words, the mechanic, the operator, the run-of-the-mill worker in industry is not thought of as having any need for a knowledge of mathematics. In some instances he may do a little calculating involving some knowledge of arithmetic. But that is not mathematics. Just some figuring that a man has to do in his daily work.

The error of such conceptions is becoming more and more apparent with the increasing need for use of mathematics in industrial work.

Progress in industry has been such, in the past several decades, that blind application of empirical formulas and methods handed down from father to son and from experienced mechanic to apprentice, is becoming more and more unsatisfactory.

This discussion could be limited to an appraisal of the mathematics needed for successful work in some of the varied activities in industry. Closely related to that, however, is the question of how this training and ability in the use of mathematics is to be acquired by people in industry. The opportunity for bringing out some of the ideas acquired on this subject is too tempting to resist. I hope, therefore, to briefly discuss both of these phases of our general topic.

Let us first of all review quickly the several types of work in industry and the knowledge of mathematics involved in each.

The engineering phase of industry will in all probability first come to mind as requiring the use of mathematical knowledge and ability. Recent developments in the aircraft industry are excellent examples of what may be expected in all industry from the standpoint of increasing need of more and more involved mathematical knowledge. The daily calculations these men have to make necessitates the use of our highest forms of mathematics. Such work as structure analysis, calculations for flutter prevention, heat transfer, pressure losses, etc., is readily associated with the mathematical methods. In this type of activity the needs and uses of mathematics are well known. The people who are training for these types of work have little if any difficulty in realizing these needs while in school, and can readily apply their academic training to practical use. We can therefore summarily dismiss the design engineer as not offering any problems in the training for, and use of mathematics in industry.

It should be borne in mind, however, that the design engineer who is concerned with the product design, still leaves some engineering work to be done. This work consists of engineering the tools with which the product is to be manufactured. In recent years tool engineering has become a very vital activity. In the future this activity will require more and more recognition as important engineering work. Our present frantic efforts to bring about maximum production have served to emphasize the importance of tool engineering. Because of the lack of

* Speech delivered before the Annual Meeting of the National Council of Teachers of Mathematics at San Francisco, California on February 21, 1942.

formal recognition of this type of engineering in the past, insufficient attention has been given to the mathematical needs and requirements of the people engaged in this work. The tool designer and the tool planner, while not using the highest levels of mathematics to any great extent, must have a good thorough basic foundation in algebra, geometry, analytical geometry, trigonometry, and in some instances differential and integral calculus. Inasmuch as many of these people have gradually developed from skilled tool makers, die makers and machinists they are often severely handicapped by a lack of mathematical ability. To supply this need is very important in present-day industry.

With this summation of engineering work in industry we are often apt to dismiss any thoughts regarding the needs for mathematics for the remainder of the people concerned. The fallibility of such a shortsighted attitude is apparent as soon as we consider the craftsmen, mechanics, and supervisors in any modern industrial organization. Among the high-skilled trades such as machinists, tool makers, die makers, loftsmen, template layout men, and so on, mathematics is as much a tool as their scale, dividers, or straightedge. In many instances these craftsmen have attained their skill from practical experience. As long as consecutive jobs are familiar enough in nature so that a rule of thumb procedure can be used, or a certain page in the handbook referred to, all is well. However, when a new project diverts somewhat from the familiar and well-traveled path a lack of sufficient mathematical ability can be disastrous. These people must have proficient ability in the use of algebra, geometry and trigonometry. When we find such people having difficulty in converting and manipulating fractions the need for mathematical training becomes painfully apparent.

Now let us consider the semi-skilled worker. The machine operator, the assembler, the installer, the metal fitter and the many other semi-skilled craftsmen engaged in the fabrication and assembly of our present-day mechanical contrivances. As these products are becoming more and more complicated there is an ever-increasing need for some mathematical ability by the persons involved in their manufacture. A simple layout for a line of rivets on an airplane, or an acute angle on a bent piece of sheet metal may require a knowledge of simple trigonometry. Whenever the person doing the work does not have the mathematical knowledge needed considerable delay is occasioned by his having to bring his problem to a superior. For these many semi-skilled workers a knowledge of mathematics including algebra, geometry and simple trigonometry is essential.

Continuing down the ladder of skills in industry we come at last to the unskilled or slightly skilled production worker. Surely this man does not need to be encumbered with the scientific and classical subject of mathematics! If we could only realize the many hundreds and thousands of hours that are lost to production activity by these workers in such mundane efforts as converting fractions to decimals, our complacency and smugness would quickly vanish. Many thousands of dollars and thousands of man-hours of effort could be saved by the use of a decimal system of dimensioning. The main obstacle to installing such a system has been the inability of unskilled mechanics to convert fractions to decimals, and vice versa. Since the study of fractions and roots is a basic part of mathematics, its need in industry is extended to nearly all the people engaged therein.

Since the need for mathematical knowledge and ability in industry has increased to a great extent, and will continue to increase, we are concerned with the means whereby the people engaged in industry can acquire this knowledge and ability. A casual consideration of this question brings forth the logical answer that, since mathematics is taught in the grade schools, high schools, junior colleges, etc.,

throughout the country, this need is met by the curricula in our school systems. However, the fact that the people who have been exposed to the mathematics courses in the schools do not have sufficient ability in the use of mathematics indicates that there is still something to be desired. Various reasons have been given for this situation. The students are not interested in mathematics. The students are not capable of acquiring such ability. The students do not have an aptitude for mathematics, and so on. These explanations would seem to indicate that while mathematics is being taught in our school systems it does not necessarily follow that mathematics is being learned in our schools. We could even go farther than that. Not only is the natural desire inherent in all human beings, for the ability to calculate their possessions by use of mathematics, dulled by the formality and "holier than thou" attitude of many mathematics teachers, but their tendency to satisfy personal egoism by spending more effort and energy in showing how clever and wise they are serves to engender a deep-rooted antipathy toward the term "mathematics" and all that it implies.

This antipathy toward mathematics is so pronounced and so evident in all our contacts with employees in industry that sometimes we are prone to wish that our schools would leave mathematics out of their curriculum. Then we would only need to give people the opportunity for learning mathematics. We would not have to combat the resentment which most employees have.

Our efforts to combat this resentment is illustrated by the fact that among some two hundred supplementary training classes in the Burbank area alone there are only two classes in mathematics. This, in spite of the great need for additional training and ability in mathematics.

This situation is not occasioned by a lack of mathematics teachers. Nor is it because we lack the classroom facilities. There are only two classes in mathematics simply because the thousands of employees that need this training will not enroll in math courses.

As a result of this avoidance of mathematics, training on this subject has to be given with every subterfuge at our command. Numerous examples can be cited. Courses in blueprint reading are copiously interspersed with problems in math. Tool Design courses devote considerable portions of their time to instruction in algebra and trigonometry. Nearly half of the time in courses in Sheet Metal and Template Layout are devoted to the study of mathematics. Of course, in every instance the word mathematics is not mentioned. It is just part of the Sheet Metal course or the Template course or whatever the subject may be. It has even been suggested that we inaugurate a course in mathematics with the title "Aeronautical Calculations" so that the stigma of the term "mathematics" would not frighten prospective students.

To merely find fault with, and bring out the shortcomings of mathematical training would be only adding one more voice to the critical chorus. From close everyday contact with many industrial employees a sympathetic attitude toward this problem has naturally resulted. From this sympathetic viewpoint several suggestions have been acquired. First of all, I would like to quote from a recent article in *School and Society* magazine by Dr. W. H. Kilpatrick, of Teachers College, Columbia University, entitled "The Philosophy of the New Education." Dr. Kilpatrick lists seven general principles in his paper. The fifth of these principles is as follows:

> "Learning goes on best in the degree that the individual himself sees and feels the significance, to his own felt needs, of what he does."

Whether this is a new principle or an idea that has been followed for many years is not for me to say. Those of you who are engaged in educational work can best testify to that. I think we all agree

that it is a good sense. How many teachers and professors of mathematics can honestly and conscientiously say that their students "see and feel the significance to his own felt needs of what he does" while they are attending classes in mathematics? My first suggestion, therefore, is that this principle of Dr. Kilpatrick be given a great deal more attention. This is much easier to do than most of us realize. At first thought it may seem difficult to make a high school student realize that the seemingly abstract studies in mathematics are somewhat useful. The teacher must first convince himself that his purpose in the mathematics class is not to prove his own superiority. Nor is it merely an opportunity for impressing young people with the difficulty and ramification of mathematical manipulation. Nor is it a means for applying mental torture to young and impressionable minds.

As soon as the instructor gets rid of these notions himself he will be in a position to impress his students with the value of mathematics to their own needs. He can then convince his students that mathematics is a tool to be used in his work or profession. It is just as much a tool as a mechanic's wrench, or the surgeon's scalpel.

In trying to overcome the fear of, and aversion toward mathematics among employees, we have often used these illustrations. A new and friendly attitude towards mathematics is quickly acquired by students when they are convinced that it is not a means of torture but a useful and necessary accomplishment.

The next step in overcoming resistance to mathematics is to eliminate the notion that it is an extremely difficult subject to conquer. Here again the instructor must first get rid of the temptation to cater to his personal egoism. He can then alleviate fears of the difficulty of mathematics. Let me cite a specific example. An employee comes to our office for advice regarding some supplementary training. A review of his work, past experiences, and training makes it apparent that this person needs to take some training in algebra and geomtry. When this is explained to the man, he counters with the remark that he would rather take some other subject because "mathematics is pretty tough." When we ask the man just what is "tough" about mathematics he is at a loss for words. However, he keeps repeating that he is wary of enrolling in such a course because he feels that it will be too difficult. It is then explained to him that of all the sciences mathematics is the easiest to master. The reason for that is simple. Of all the sciences mathematics is the most exact. Two plus two equals four, whether it is three o'clock in the afternoon or five o'clock in the morning; whether the temperature is 110° in the shade, or 40 below zero; whether we are atop a 7,000 foot mountain peak, or a mile below sea level. In other words, there is no guess work in mathematics. No experiments to be affected by extraneous conditions. The mastery of this science is therefore a simple application of basic fundamentals. Anyone following these fundamentals step by step, slowly, if his mentality is slow to absorb such knowledge, rapidly, if his mentality is capable of absorbing rapidly, can acquire mathematical knowledge and ability. In any event, because of its very nature mathematics is the easiest subject to study. Such an explanation generally throws a new light on the subject. The little-known, much feared, phantom becomes a familiar and negotiable obstacle. Fear is eliminated and the battle is more than half won.

When we have established, to the satisfaction of the students, that mathematics is useful to his own needs and that mathematics is easily mastered, it remains but to follow with the next logical step; that is, to make mathematical instruction practical, useful, and easy to absorb. In this respect we can do two things. First, eliminate some of the moss-covered rituals that have been retained by textbook authors and instructors. The instructor that marks a solution wrong because the student had neglected to add Q.E.D. at

the end of the problem is not only serving no useful purpose but is actually obstructing mathematical training. The instructor who is more concerned with the proof of the Pythagorean theorem than its use and practical application may be teaching "classical mathematics," but he is greatly reducing the number of students who are learning mathematics.

The problems assigned to students must be real practical applications of the theory being studied. Here again our best starting point is the instructor himself. If the instructor does not have the practical attitude or if he takes the simple expedient of slavishly following the textbook, the applications by means of problems can become stilted and unreal. The young and impressionable student follows the tendency of making himself the hero of all imaginery experience. Let me cite a specific example. I can recall a trigonometry instructor being quite pleased with a "practical" problem in trigonometry. This problem consisted of finding the height of a tree by measuring its shadow at a certain time of day. The angle of the sun at that time of day was given, as was the length of the shadow. It was then required to calculate the height of the tree. The problem was assigned on a Monday and was due the following Wednesday. Over the previous weekend and during the remainder of the week it rained incessantly. Not even a glimpes of the sun had been possible during that time. In working the problem, I envisioned myself as camped under a tall pine tree with a watch in one hand and a measuring tape in the other waiting for the moment when the sun would appear so I could dart out and measure the length of the shadow. The whole thing seemed phantastic for I knew, and the instructor knew, that the practical way of measuring the height of the tree would have been to set up a transit at some measured distance from the tree, measure the angle to the top, and from that information calculate its height. Here we have an example of the so-called practical problem concocted by an instructor without the practical background. His being quite proud of this problem further tended to reduce my respect for his ability. How can the student feel the need for acquiring mathematical ability when the examples given him are not borne out by practical experiences? This same idea was quite dramatically illustrated in the recent motion picture "How Green Was My Valley." It may be recalled that the student in this story was being helped by his parents in solving a problem that went somewhat as follows: *A pours 10 gal. of water per minute into a bathtub of a certain given size. B pours 5 gal. of water per minute into the same tub. Through holes in the bathtub, water is leaking out at the rate of 4 gal. per minute. How long will it take for the tub to be filled* "
The mother in the story dismisses the whole subject as "stuff and nonsense." "Who ever heard of a bathtub full of holes?" It is rather tempting to continue on with examples such as these; however, the two cited should be sufficient to illustrate the point.

In summation, let me point out that the need for mathematical training in present-day industry is greater than ever before. This need is increasing at a very rapid rate. All indications are that it will continue to increase.

To meet this need a great deal can be accomplished by a progressive open-minded attitude in our schools. We can first teach mathematics as a useful and necessary tool in everyday life, as well as industry. We can teach it (to quote our Dr. A. T. Helbing) "not as art for art's sake but as art for Pete's sake."

We can stifle any egotistical temptation to emphasize the difficulties in mathematical manipulations and teach it as an exact, easily assimilated science.

We can teach mathematics on a practical basis so that our young people will not only be taught math in school but will also learn math in school. In this manner we will be rendering a great service not only to industry and to the people engaged therein, but to the country as a whole.

The High School Training Most Desirable for Prospective Students of Engineering*

By Tomlinson Fort
Lehigh University, Bethlehem Pennsylvania

I gotta love for Angela,
  I love Carlotta, too.
I no can marry both o' dem,
  So w'at I gona do?

Oh, Angela ese pretta girl,
She gotta hair so black, so curl,
An' teeth so white as anytheeng.
An' oh, she gotta voice to seeng,
Dat mak' your hearta feel eet must
Jomp up an' dance or eet weel bust.
An' alla time she seeng, her eyes
Dey smila like Italia's skies,
An' makin' flirtin' looks at you—
But dat ees all w'at she can do.

Carlotta ees no gotta song,
But she ees twice so big an' strong
As Angela, an' she no look
So beautiful—but she can cook.
You oughta see her carry wood!
I tal you w'at, eet do you good.
W'en she ees be som'body's wife
She worka hard, you bat my life!
She nevva gettin' tired, too—
But dat ees all w'at she can do.

Oh, my! I weesh dat Angela
  Was strong for carry wood,
Or else Carlotta gotta song
  An' looka pretta good.
I gotta love for Angela,
  I love Carlotta, too.
I no can marry both o' dem,
  So w'at I gonna do?

T. A. Daly

The dilemma in which this stalwart son of Italy found himself need never be confronted by the teacher of secondary school mathematics so far as the training of prospective students of engineering is concerned. It is my belief that the mathematics to be taught to the would-be engineer is substantially the same as that to be taught to anybody else and we are not confronted by the necessity of neglecting our engineers in order to cater to the wants of some other substantial group of students.

Now what do college teachers of mathematics wish first and foremostly that the high school teachers do? It is: Do not send us poor students. A student may be poor for at least three reasons: (a) lack of natural ability, (b) insufficient preparation, (c) faulty preparation. I do not list laziness or a wrong attitude toward his work. Every teacher prizes brains more than training. Give me a smart boy and I can give him training with surprising quickness. Thus in my opinion most of us teachers of college mathematics want smart boys more than anything and would like the high schools to act as a sieve so that those who get college certification in mathematics really have the brains to do the work. I rather think that the high school teachers do as good a job as we college teachers do. We graduate some who are pretty weak brothers. But I plead for high standards all along the line and there is one definite place where high standards can be set, namely at college certification. I trust the knowledge of a student gained by the high school teachers through daily association and frequent testing far more than I do any single examination set by the college, by the College Entrance Board, by the American Council or any other agency. Examinations are fair in prospect but we know too little about how to prepare them. Alone they are at best none too good an instrument.

The next thing to the brains with which a man is born is training. I hold that sound mathematical training aims at preparing the student to face a mathematical situation with confidence, to think quickly and accurately and to execute neatly, to visualize geometrical or mechanical configu-

* Address delivered before the Christmas meeting of the National Council of Teachers of Mathematics at Bethlehem, Pa., on December 30, 1941.

rations, to plan a solution, to write a good hand and last, but far from least, to compose accurate concise English. These abilities are, in my opinion, more important for progress in mathematics than particular subject matter. Not that subject matter is without importance. But the former qualities, which go to make up what I call mathematical maturity, I regard as more important for the success in college mathematics for which we are striving. I am calling for good teaching with a thorough grounding in fundamental mathematical processes.

Subject matter! Of course, we must have subject matter. The mathematics curriculum from the beginning to the end is a matter of constant prerequisites. We must have the tools of our trade. One can not progress without arithmetic, algebra, geometry, trigonometry. I do not have a great deal to offer here. I regard the usual courses as soundly planned. I do not have much sympathy with the tendency to eliminate everything which it is thought the student will not frequently use, or everything which can not be found in use in the next more advanced book. Not only does this result in the progressive weakening of the whole course, but we must remember that in order to use mathematics of Grade x one must have studied far beyond Grade x. There must be much practice on this more advanced work if the problems of so-called practice are to be faced with confidence, skill and true knowledge.

In detail. I believe in solid geometry. We can get on without it beyond simple nomenclature and mensuration, but nowhere else does the boy get training in space visualization. Also I just believe in geometry. Only there in the whole of his school course does the student encounter systematic proof. In trigonometry, beyond the right triangle, I should decrease the time given to numerical solutions and, correspondingly, increase the study of the functions and of radian measure. Elementary algebra is fundamental and I rather emphasize it as against even intermediate algebra. But I believe in algebra: elementary, intermediate and advanced.

Now let me make a quotation from "Alice in Wonderland."

'And how many hours a day did you do lessons?' said Alice, in a hurry to change the subject.

Ten hours the first day,' said the Mock Turtle; 'nine the next, and so on.'

What a curious plan!' exclaimed Alice.

That's the reason they're called lessons,' the Gryphon remarked, 'because they lessen from day to day.

This was quite a new idea to Alice, and she thought it over a little before she made her next remark.

Then the eleventh day must have been a holiday?

Of course it was,' said the Mock Turtle.

And how did you manage on the twelfth?" Alice went on eagerly.

That's enough about lessons,' the Gryphon interrupted in a very decided tone; 'tell her something about the games now.

If the boy is to be an engineer do not lessen his study of mathematics. You have no idea how much we suffer from the boy who comes to our classes but has had no school mathematics for one or two years. Keep him at it! How we dread the rusty student! If he is to be an engineer make him take his courses over if you have no new courses for him. Don't let his acquired mathematical skills be dissipated from lack of use.

Let the prospective student of engineering know what he is up against. Do not send to the university the student who is suited only for the trade school. A boy likes to play with radio. Should he be an electrical engineer? It is not sufficient reason. Let him know so well as you can the arduous mathematical course of the electrical engineer. Perhaps the trade school is the place for him. A radio repair man or radio store manager is a worthy occupation for which the trade school may be the best preparation. Perhaps he should go into some other business with radio as a

hobby. Vocational guidance is difficult and uncertain, but it is greatly needed. We teachers must not be afraid to tell students what we think.

We must interest our students or they will not study. But remember—the college teacher of mathematics is not a teacher of engineering and, if he so elects himself, he is apt to be a poor teacher of engineering. His job is to teach mathematics—applications yes—but as a side issue. Mathematics is the real job. Even more is this true of the teacher of secondary school mathematics. Mathematics is the job. Applications to make it interesting and to train the student in that very important thing, the translation of the story problem into algebra or into figures, but we, all of us, are teachers of mathematics. It is a job that I am proud to do, that we all are proud to do. I think that we do a good job. I am not rating anyone along the line as doing a better or a poorer job than anyone else. Remember the man who takes your students next. The high school teacher must think of the teacher of freshmen. The teacher of freshmen must remember the teacher of sophomores. Let us send along able, well-prepared students and let us not be afraid to turn back the others.

Now so much for what the collegiate teachers of mathematics want. What, in general, does the faculty of the College of Engineering want? What do the teachers of engineering itself want?

Firstly, they do not want engineering itself taught in the secondary schools. I think, however, that all, or almost all teachers of engineering would rate a sound and thorough training in high school mathematics at the top or about the top of the list of what is important in high school. The questions of what to teach engineers and what qualities are desired in the engineer have been greatly discussed in the Journal of Engineering Education and at meetings of the S.P.E.E. We hear much of our uneducated engineers and most leaders in engineering want them educated. This reflects the attitude of the engineering profession itself in its desire to assume a more prominent part in society in the large, a post of leadership outside its profession, in the management of industry, in business, in politics, in the church.

In order for a man to take this position he must have what I describe as a good general, or cultural education, education for life outside his technology. There is but little time in college for the student of engineering to get this type of work and, if it is to be had at all, most of it must be had in high school. But few students of engineering study any history in college, none study enough English composition and but few enough English literature. Almost none study botany. None study Latin or Greek, yet there are many of us who hold that the person without a smattering of Latin and some knowledge of ancient history never really understands many aspects of the world in which he lives and has locked to him forever a true appreciation of a large part of the world's great literature. My plea, then, is for the high schools to help the helpless colleges with this problem of the uneducated engineer. I plead here for the old-fashioned studies, Latin, mathematics, history, composition and literature, for the old-fashioned academy as against the vocational high school. There is no firmer basis for health, enjoyment, leadership, good citizenship. Let vocational work be a superstructure. Let engineering stand on a high school foundation of Latin, history, English composition, English literature, art appreciation, music and mathematics. Most sciences will come in college. Let the colleges primarily teach mathematics, science and technology to students of engineering. Let them uphold the ideal of technical and scientific excellence. However, extreme specialization probably belongs even later still, in the graduate school.

The general education of which I have spoken is the proper foundation for any

career. So I return to my opening idea. As there is not much difference between the mathematics to be taught to students of engineering and to any others, so there is not much difference between the whole education that high schools should give a prospective student of engineering and that which they should give any other student of first class mental ability, and I plead that only such be sent to college at all. I think, also, that these are the ones that are worth most. Don't neglect the poor student, but remember that where your teaching really counts is the good student. Don't plan your curricula so that you cater to mediocrity only. Don't neglect the highly intelligent for those who will never be other than mediocre. It seems to me that we teachers talk much about the fundamentals of education, but don't do much about them. As framed by Omar Khayyam,

> Myself when young did eagerly frequent
> Doctor and Saint, and heard great argument
> About it and about: but evermore
> Came out by the same door where in I went,

Let us not be like that.

Now a final word. Send us to college smart, well-educated boys, particularly in mathematics.

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W. D. CAIRNS, Secretary-Treasurer

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Geometry for All Laymen

By HAROLD FAWCETT
Ohio State University, Columbus, Ohio

DURING the summer it is my privilege to guide the thinking of twenty-two young people as together we explore the inviting opportunities offered in the study of demonstrative geometry. On meeting these boys and girls for the first time about two weeks ago I discussed with them the nature of their previous experiences in mathematics. I listened while some who were more talkative than others told of their work during the preceding years and in their reports I saw reflections of what might be recognized as arithmetic and informal geometry. Through encouragement and an occasional leading question I learned that all of them had had various degrees of experience with a course in elementary algebra. One girl did insist that she had never had such a course but when assured by the other members of her class that she had taken it with them she replied "Oh! I remember now and you are right. I was in there." I learned that four of the twenty-two students really enjoyed their algebra while six thought that the course had been of some value to them although they were unable to say just what these values were.

Against this background I asked them what it was they hoped to accomplish through their summer program. Why did they decide to study demonstrative geometry which was not required for graduation and which so many of their friends reported as being "very difficult." What values did they expect to achieve and what reasons did they have for taking this work. I asked each student to state these reasons in writing, suggesting that they be ranked in order of decreasing importance. A study of these statements revealed that there are three main purposes which these young people hope to realize through the work they have undertaken. Each of them feels that in some way or other the study of demonstrative geometry will improve the quality of his thinking and twenty-one of the twenty-two ranked this first in order of importance, one student placing it last. The other reasons given were increased geometric knowledge and credit toward graduation, these being equal in importance as far as these particular students are concerned.

I wish now to emphasize the point that here are twenty-two students, most of whom had derived no real satisfaction from their study of mathematics but all of whom had returned voluntarily to summer school with the idea that the study of demonstrative geometry would in some way or other "improve the quality of their thinking." Are they to be disappointed in the outcomes achieved? Can the quality of their thinking be improved by the kind of academic experiences they are now having? If I had no faith that that desirable result were possible I would resign at once and lend my active support to those who recommend that the study of demonstrative geometry be reserved for the intellectual aristocracy who do not want their thinking improved but who find comfort in the fact that geometry may be used as the key that unlocks the door to the college of their choice. If that result were not possible then the study of geometry becomes nothing more than a fact-gathering enterprise and as useful as these facts are to many people they can be learned and are learned without logical proof. If this purpose, which ranks first in the desires of these young people, cannot be achieved then the claims made for demonstrative geometry are nothing more than bedtime stories which have lulled our sleepy intellects into peaceful and permanent slumber. Let us listen to the following quotation from the work of a distinguished teacher:

Geometry achieves its highest possibilities if, in addition to its direct and practical usefulness, it can establish a pattern of reasoning; if it can develop the power to think clearly in geometric situations and to use the same discrimination in non-geometric situations; if it can develop the power to generalize with caution from specific cases, and to realize the force and all-inclusiveness of deductive statements; if it can develop an appreciation of the place and function of definitions and postulates in the proof of any conclusion, geometric or non-geometric; if it can develop an attitude of mind which tends always to analyze situations, to understand their interrelationships, to question hasty conclusions, to express clearly, precisely, and accurately non-geometric as well as geometric ideas.[1]

That statement comes from the pen of Dr. H. C. Christefferson, past president of the National Council of Teachers of Mathematics. He is but one of a long line of distinguished witnesses who have likewise testified to the educational possibilities inherent in the study of geometry. Are these desirable values and can these possibilities be realized? That they are highly desirable is indicated by the fact that if there is one general purpose to which the American public school is dedicated it is to "improve the quality of thinking" of the students for whom it was called into being. That they are being realized to an ever increasing degree is supported by such significant studies as those made by Mr. Gilbert Ulmer of the University of Kansas and Mr. R. E. Gadske at Northwestern University. In each of these experimental studies a definite attempt was made to improve the quality of pupils' thinking by using geometric content as a medium through which to study principles of reflective thinking. Eighteen teachers participated in Mr. Ulmer's study and 1239 pupils were involved. Here are his conclusions as reported in the September, 1939, issue of the *Journal of Experimental Education:*

The results of this study indicate that it is possible for high school geometry teachers, under normal classroom conditions, to teach in such a way as to cultivate reflective thinking, that this can be done without sacrificing an understanding of geometric relationships, and that pupils at *all* I.Q. levels are capable of profiting from such instruction. The results also indicate that even what is commonly regarded as superior geometry teaching has little effect upon pupils' behavior in the direction of reflective thinking unless definite provisions are made to study methods of thinking as an important end in itself.

Mr. Gadske's study in which twelve teachers participated and which included 659 students lends very substantial support to these conclusions and they are further supported by the results of my own more limited experience.

Now I am not unmindful of the fact that many other kinds of experiences offer potentialities for improving the reflective thinking of our students and while his total educational experience in the secondary school should contribute and doubtless does, to some extent, contribute to the realization of this very desirable purpose it is my judgment that in the subject matter of demonstrative geometry are possibilities offered by no other subject. In this area the concepts considered and the ideas studied are devoid of strong emotional content. The student's native ability to think is not stifled by prejudice or bias. Under the guidance of his teacher he can enter into such significant learning activities as creating and refining his own definitions, critically examining the assumptions he is willing to accept and studying their implications. He "learns by doing" and there are potential refinements about this kind of learning and "doing" which in my humble judgment are not possible in any other area of his secondary school experience. He slowly becomes conscious of the fact that the so-called "laws of geometry" were not divinely revealed to some mathematical Moses in a Mt. Sinai experience but that rather they are the logical outcomes of the definitions and assumptions to which he, himself, has agreed. He learns what it means to "challenge an assumption" for in the exercise of his free intellectual sovereignty he can change an assumption

[1] H. C. Christefferson, *Geometry Professionalized for Teachers*, p. 28. Geo. Banta Publishing Co., Menasha, Wisconsin, 1923.

to suit his own individual ideas and trace the inevitable effect of such change on the conclusions which depend on that assumption. He has the stimulating experience of seeing a method of thought applied to idealized concepts and, in the words of Professor Keyser, "without that ideal, thinking is without a just standard for self-criticism; it is without a light upon its course; it is a wanderer like a vessel at sea without a compass or star."[2] This need is met by the logical rigor of mathematical proof and according to the recent report of the Joint Commission published in the Fifteenth Yearbook of the National Council of Teachers of Mathematics "mathematical methods set up an ideal, a norm, a pattern which, though not attainable in all the activities necessary and proper to man, serve as a relentless spur to improvement."[3]

There is then some justification for the faith of these twenty-two young people that through their summer work the quality of their thinking will be improved and if this very desirable result is not achieved the greater part of the responsibility for such failure will, in my judgment, rest upon their teacher and will in no sense be a function of the subject they have studied. The outcome they desire is not a necessary outcome. It is possible to teach demonstrative geometry in such a manner as to give the students the idea that Euclid entered into some kind of a partnership with God from Whom he received the axioms and postulates which made it possible for him to lead an uncritical following into the promised land of absolute truth where all creative thinking ceases. It is possible to chant so seductively about the "self-evident truths" which gave birth to Euclidean space that the student will come to recognize them as the sacred laws of the Almighty and to tamper with them is like tampering with the ten commandments. Any such emphasis tends to deaden the critical faculties of our students and to stifle effective achievement of the very values claimed for the subject. The students come to believe that "Euclid discovered all there is to know about geometry. His conclusions are all in the book and they are absolutely true. We will memorize them and there will be nothing to worry about." Is it any wonder then that thirty-nine years ago John Perry said in the Report of the British Mathematics Association:

> I believe that men who teach demonstrative geometry and orthodox mathematics generally are not only destroying what power to think already exists, but are producing a dislike, a hatred, for all kinds of computation and therefore for all scientific study of nature, and are doing incalculable harm.

As mathematics teachers our behavior has suggested a rather naïve faith in the assumption that since demonstrative geometry offers inviting possibilities for the development of reflective thinking that outcome is necessarily achieved through the study of the subject. The results of the past do not tend to validate this assumption and I have often wondered whether those of us who teach geometry reflect the quality of thought which we claim the study of the subject will tend to develop in our students. May I again read a sentence from the conclusions of Mr. Ulmer's study:

> The results also indicate that even what is commonly regarded as superior geometry teaching has little effect upon pupils' behavior in the direction of reflective thinking unless definite provisions are made to study methods of thinking as an important end in itself.

This means, then, that if I am to be faithful to the responsibilities I have assumed in connection with my summer laboratory class I must give attention to such matters as methods by which conclusions are established, the necessity for clearly defined terms and their effect on the conclusion, the function of assumptions, the

[2] C. J. Keyser, "The Human Worth of Rigorous Thinking," *The Mathematics Teacher*, Vol. XV, No. 1, 1922, pp. 1–5.

[3] The Joint Commission, *The Place of Mathematics in Secondary Education*. Fifteenth Yearbook of the National Council of Teachers of Mathematics, p. 8.

meaning of proof, conditions which make generalizations safe and the like. To do this I may not be able to cover all the theorems called for in the course of study and I may find it difficult to be on page 116 by the end of the summer session. However, I can so guide the thinking of these young people that all the important ideas of geometry can be examined and at the same time I can create an atmosphere in which the study of geometry may achieve "its highest possibilities." Furthermore, if one wishes to be protected against the indictment that he is not covering a sufficient amount of subject matter he can be guided in the selection of the propositions studied by the results of such useful researches as those of Christofferson,[4] Fagerstrom[5] and Pickett.[6]

Now these students are kept informed of administrative rulings through a bulletin which, whenever issued, is read to them by a representative of the administration. A consideration of procedures in connection with tardiness occurred in the first of these bulletins and in the course of the discussion there was the statement that "habitual offenders" would be required to leave school. I used that statement as a point of departure for our work as it offers an excellent illustration of the importance of definition in a context which is of real concern to these young people. After commenting on a number of points related to the tardy problem I asked how an "habitual offender" was determined. This conclusion stimulated various kinds of responses but in reality none of these students knew how many times they could be tardy before they were classified as an "habitual offender." We then found out from the administration how an "habitual offender" was defined and we had our first impressive lesson as to the importance of clearly defined terms. Numerous other illustrations were then supplied by the students and among them was the statement of Tom Dewey that he was "in favor of proper aid to the allies." This led to an increasingly warm discussion as to how "proper" should be defined and when the smoke became too thick and it became evident to all concerned that no agreement could be reached we turned the direction of the discussion into a more fruitful field where "the concepts considered and the ideas studied are devoid of strong emotional content" and "the students' native ability to think is not stifled by prejudice or bias."

Out of a number of suggestions as to the concepts which need to be defined before we could begin to build a theory about the space in which we live we selected "angle" and through an attempt to define this concept the need for a number of undefined or primitive terms was established. In a similar manner the necessity for a number of initial assumptions or unproved propositions will be established and this same pioneering and exploratory spirit will permeate the remainder of our work together. We shall practice the philosophy of E. H. Moore,[7] expressed in his great address as retiring President of the American Mathematical Society thirty-eight years ago:

> Why should not the students be directed, each for himself to set forth a body of geometric fundamental principles, on which he would proceed to erect his geometric edifice? This method would be thoroughly practical and at the same time thoroughly scientific. The various students would have different systems of axioms and the discussions thus arising naturally would make clearer in the minds of all precisely what are the functions of the axioms in any theory of geometry.

In an earnest effort to translate this philosophy into practice we shall write our own definitions and we shall make our own assumptions as we progress in the building of our theory. The first state-

[4] Christofferson, H. C. Op. cit.

[5] Fagerstrom, W. H. *Mathematical Facts and Processes Prerequisite to the Calculus*, Bureau of Publications, Teachers College, New York, 1933.

[6] Pickett, Hale. *Proofs and Solutions of Exercises Used in Plane Geometry Tests*. Bureau of Publications, Teachers College, New York, 1938.

[7] Moore, E. H. "On the Foundations of Mathematics" First Yearbook, pp. 32–57.

ments of these definitions and assumptions are likely to be loose and ambiguous and we shall call upon the intellectual resources of the entire twenty-two students in clarifying and refining them. We shall study the implications of these definitions and assumptions and there will be continued emphasis on the function of these basic ideas in establishing any conclusion by logical proof. The circumstances which gave birth to non-Euclidean geometry will be used to illustrate how a conclusion may be affected by a change in one of the basic assumptions on which it depends and we shall consider what is meant by the "truth" of a conclusion. Our investigations will be extended to include both two and three dimensional concepts and many useful properties of space will be established. The detection of implicit or tacit assumptions will be encouraged and numerous exercises will be provided in which the student will be asked to give explicit expression to the assumptions used in the given proof.

It should be recognized, however, that to study situations which involve only the idealized concepts of mathematics is to limit the extent to which the kind of thinking developed may be used. If this kind of thinking is to be applied in non-mathematical situations such situations must be considered during the learning process. In a stimulating and suggestive article on "The New Psychology of Learning" published in the Tenth Yearbook of the National Council, Raymond H. Wheeler writes:

> No transfer will occur unless the material is learned in connection with the field to which transfer is desired. Isolated ideas and subjects do not integrate. Learning is not bond forming. It is an orderly and organized process of differentiating general grasps of situations with respect to experience.

Transfer is secured only by training for transfer and teachers of mathematics can no longer expect that the careful study of ninety or more geometric theorems will alone enable their students to distinguish between a sound argument and a tissue of nonsense. If this result is to be achieved the methods of thought used so effectively in connection with geometric content must be extended to situations of a non-mathematical character. The student, for example, may be asked to examine the Declaration of Independence and to select the assumptions which the signers of this document considered fundamental to democratic government. His attention will be directed to the quality of argument found in political speeches, editorials, advertisements and the like and he will be asked to analyze arguments of this type, selecting the words and phrases on which the validity of the argument depends as well as the stated and unstated assumptions which are essential to the conclusion reached. It is in this manner that definite provisions will be made to "study methods of thinking" and to use them "in connection with the field to which transfer is desired." Through such procedures explicit expression is given to the suggestions in the report of The Committee on Geometry such as:

"Actually do transferring from geometry to other fields."

"Apply forms of reasoning to non-mathematical situations."

"Point out the parallel between thinking in geometry and in other fields."

"Bring in illustrations to show the place of logical thinking in life."

And while I am not sure just how many geometric theorems may be established during the eight week period I know that the mathematical souls of these young people will not be entirely lost. This is not the first time that students have been exposed to procedures of this sort and if past experience is any safe criterion by which to judge their continued record in this area is likely to reveal a rebirth of interest as well as increasing insights into the methods of mathematics.

May I point out, however, that I am not thinking only of those students whose promise of achievement in the field of mathematics is especially inviting. My

responsibility does not end with them. I am thinking of all the students who move through the classrooms of our secondary school and I am considering the nature of a course in demonstrative geometry which has potential promise of making a genuine contribution to the education of every one of these young people. I would not for a moment belittle the importance of the values served through a comprehensive knowledge of geometric facts but I would also point out that a relatively small number of the 130,000,000 people in the United States have experienced whatever satisfaction there may be in knowing that the diagonals of a parallelogram bisect each other or that the medians of a triangle are concurrent. It is also true that the great majority of these people will never be faced with the necessity of analyzing the evidence presented in support of a mathematical theorem but it is just as true that no thoughtful citizen of a democracy can avoid the necessity of examining the evidence in support of the great variety of conclusions he is pressed to accept. In the long history of human thought there has probably never been a time when the ability to detect the hidden assumptions in an argument and to recognize the "weasel words and phrases" used was more to be desired than at present and, unfortunately, problems of this sort are not limited to those whom inheritance has favored with an intelligence quotient of more than 110. They come to the intellectually rich and poor alike and as teachers of mathematics we have a responsibility for improving the reflective thinking of *all* our people if we would be faithful to the obligations of our profession. Through such experiences as those I have discussed, it is my opinion that this responsibility will be met. Mr. Ulmer reports "that pupils at all I.Q. levels are capable of profiting from such instruction." My own experience varifies this conclusion and it is my humble judgment that we will be meeting our responsibilities more fully if the ideas suggested should become the basis of a course in "Geometry for all Laymen."

For I dipt into the future, far as human eye could see,
Saw the Vision of the world, and all the wonder that would be;

Saw the heavens fill with commerce, argosies of magic sails,
Pilots of the purple twilight, dropping down with costly bales;

Heard the heavens fill with shouting, and there rain'd a ghastly dew
From the nations' airy navies grappling in the central blue;

Far along the world-wide whisper of the south-wind rushing warm,
With the standards of the peoples plunging thro' the thunder-storm;

Till the war-drum throbb'd no longer, and the battle-flags were furl'd
In the Parliament of man, the Federation of the world.

There the common sense of most shall hold a fretful realm in awe,
And the kindly earth shall slumber, lapt in universal law.

—*Locksley Hall*—1842
TENNYSON

"THE ART OF TEACHING"

The High School Mathematics Club

By ORVILLE A. SULLIVAN
Oak Park and River Forest Township High School, Oak Park, Illinois

IN THE course of a typical day's classwork the mathematics teacher is asked one or more questions such as, "How can there be any practical use for numbers which are imaginary?" "Could the surveyor measure this distance in any other way?" "How many ways are there for proving the Pythagorean theorem?" "How is *pi* obtained?"

Certainly many of the questions can be and should be answered or worked out in connection with the classwork from which they arise; however, class time is not always available to give all such subjects a thorough and interesting discussion.

Scores of questions similar to the above pass through the alert pupil's mind as his mathematical knowledge broadens. Too often the pupil feels that the class and teacher are too busy with the work at hand to be sidetracked with some question not primarily on the subject. And too seldom does a pupil take the initiative to approach the teacher with his questions after class, so they remain unanswered. Herein lies an important challenge to us mathematics teachers to find a time and a way to stimulate this budding interest and curiosity rather than let them die a gradual but certain death.

Here a mathematics club can be of invaluable assistance in motivating the pupil's curiosity and imagination in mathematics. Knowing that the club is designed for the purpose of following up the special interests of its members, the pupil feels perfectly free to present his problem to the group where it may be subdivided, if necessary, and given to several pupils who have a common interest in that type of problem.

Our own club has an extensive list of subjects which have been or will be used in making up the club programs. A large number of these have come directly from the classroom and others have grown out of club activities. A few random examples are.

What is a limit?
Mathematics in war.
The history of zero.
Explanation of the adding machine.
Shortcuts in computation.
Mathematical quiz program.
Lecture by someone using mathematics in business.
Mathematical plays.
Trick problems.
Finding errors in questionable solutions.
Mathematical games.
Origin of Arabic numbers.
Approximate trisection of an angle.
The decimal point.
Our number system on a base other than ten.

Any of these can be made into mathematics club programs which have abundant interest for the mathematics pupil.

Many of these can be worked out as a project with the entire club working together in such a way as to make them real fun and at the same time informational. For example: in our own club, a question arose concerning the tossing of coins. Here was potential interest in a phase of probability. At a later meeting we divided the club into smaller groups, gave each group ten pennies, a drinking glass and a flat pan. Instructions were to

tabulate the results on graph paper according to the number of heads coming up each time the pennies were thrown and to draw any conclusions that seemed in general to be true as the experiment progressed. Gradually the normal frequency curve began to form for each group, and when the combined results, about a thousand throws were graphed on a large sheet, an interesting and valuable discussion followed, broadened by some who had been asked to do special reading on the subject preceding the experiment.

In answer to numerous questions concerning the much talked of Theory of Relativity of Einstein the club secured a movie in which the underlying principles of the theory are very clearly and interestingly illustrated, entirely within the scope of understanding of the high school pupil. Any school with a projector would do well to secure this movie.

Spice and variety are added to each program by a "trick problem of the week," an "amazing problem," a stunt, writing the minutes in the form of poetry, a critic's report, and so on. Examples of "trick" problems are:

1. If one regular candle can be made by melting 6 candle stubs, how many candles can be made from 36 stubs? (Ans. 7)
2. Two men contract to dig a 100 yd. ditch for $100. One end is to be deeper than the other so the man starting at the deep end gets $1.25 per yd., and the man starting at the shallow end gets $.75 per yd. ($2.00 for 2 yd.) They dig until they meet. Each earns $50. How many yds. does each dig? (Different methods of solution give different answers.)

Examples of "amazing" problems are:

1. A clerk was being hired. He was given his choice of starting at a salary of $1000 per year and getting a raise of $20 per year, or of starting at the same salary but getting a raise of $5 every half year. If he is paid only once every 6 months which offer should he take?
(The $5 raise.)
2. Suppose that it takes 4½ hours to drive from Chicago to Indianapolis. If a bus leaves Indianapolis every hour, how many buses would a motorist meet traveling from Chicago to Indianapolis? (Ans. 9.)

Refreshments in the school cafeteria at the end of each meeting are paid for from membership dues. With something new and interesting for each program, combined with the opportunity for social contacts, centered around refreshments, the club is much more than "just another mathematics class."

Inquiry shows that an extremely small percentage of schools have a mathematics club, or do any kind of extra-curricular work of a mathematical nature. The possibility or necessity of organizing a club seemingly has received little consideration from many mathematics teachers. This may be due mainly to the fact that they have not seen one already set up and working and therefore have not been convinced of the benefits of a mathematics club to their curriculum, or they may doubt that a real and lasting interest can be obtained in a club of such academic nature. As an assurance in this respect I might say that our own mathematics club is now completing its twenty-eighth year. Starting with a membership of about thirty, it has grown until this year we had a hundred applications, all juniors and seniors with a "B" average or better in mathematics.

There is no doubt in my mind that this interest and enthusiasm developed in the mathematics club motivates interest in the classroom and helps establish in the mind of the pupil a much more receptive attitude toward both its study and its teaching. Attaining results like these, the mathematics club is a valuable teaching aid, and more important still a valuable aid to learning.

EDITORIALS

May Expirations

As stated on several previous occasions, The Mathematics Teacher is put to a great deal of unnecessary effort and expense because many of the members of *The National Council of Teachers of Mathematics* whose subscriptions expire in May each year do not renew their memberships promptly. On account of the unusual value of the articles in this issue and the desire on our part that no member may miss it, we are mailing this number to all members, even though their dues are not yet paid. We hope that such dues will be sent in immediately as we cannot afford to mail out any further copies after the October issue to members who have not renewed their subscriptions.

W. D. R.

Arithmetic in the Schools

Some of the leaders in the field of mathematical education feel that because of the attempt to postpone the teaching of arithmetic all along the line in the elementary school, there will be no time or place for teaching informal geometry in the seventh grade of the junior high school and very little, if any, in the eighth grade. There are two reasons why this need not and should not be the case. In the first place, with proper reorganization and improved methods of teaching, we ought to be able to teach the fundamental operations as such, with integers and fractions, both common and decimal, by the end of the sixth grade, so far as the needs of the rank and file of American citizens are concerned. This may mean that some of the traditional content may have to be modified or omitted altogether in the first six grades. Secondly, and more important perhaps, there is a great deal of the applications of arithmetic that we now attempt to teach in the junior high school, particularly in the eighth grade, like the inverse cases of percentage, that is beyond the understanding of many pupils who have to study the subject, and even to some of their teachers. Such material and some of the more recent additions should be included in a course in Social Economic Arithmetic and should be taught in the senior year of the high school, as was recommended by the Joint Commission in the Fifteenth Yearbook of *The National Council of Teachers of Mathematics.* This will leave ample time for a unit of informal geometry in the junior high school.

W. D. R.

The Annual Meeting in 1942

As things now stand, it is planned to hold The Annual Meeting of The National Council of Teachers of Mathematics at the Hotel Chase in St. Louis on February 27th and 28th, 1942. Members who plan to attend this meeting should write at once for reservations. It is of particular importance that mathematics teachers who can possibly attend this meeting do so, because of the great importance of teaching mathematics during the present emergency. The program for the meeting will appear in a later issue of The Mathematics Teacher.

W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
The Bronx High School of Science, New York City

The American Mathematical Monthly

March, 1942, Vol. 49, No. 3.

1. Thébault, V., "The Adams Sphere," pp. 170–173.
2. Goormaghtigh, R., "A Study of Quadrilateral Inscribed in a Circle," pp. 174–181
3. Bruce, J. M., "Approximation to a Circular Arc," pp. 184–185.
4. Smiley, M. F., "A Proof of Sturm's Theorem," pp. 185–186.
5. "The Letter of Admiral Nimitz," pp. 212.

April, 1942, Vol. 49, No. 4.

1. Kossack, C. F., "Mathematics Placement at University of Oregon," pp. 234–237.
2. Hammer, P. C., "Mantissa and Characteristic," pp. 245–246.
3. Lange, Luise, "Geometrical Aspects of the Power Function," pp. 248–249.
4. "Pre-training of Aviation Cadets," pp. 274–276.

May, 1942, Vol. 49, No. 5.

1. Kline, J. R., "What is the Jordan Curve Theorem?," pp. 281–286.
2. McCoy, N. H. "Remarks on Divisors of Zero," pp. 286–295.
3. Brown, B. H., "The Euler-Diderot Anecdote," pp. 302–303.
4. Dorwart, H. L., "Values of the Trigonometric Ratios of $\pi/8$ and $\pi/12$," pp. 324–325.
5. Coolidge, J. L., "A Simple Geometrical Paradox," pp. 325–326.
6. Householder, A. S., "The Addition Formulas in Trigonometry," pp. 326–327.
7. Condon, E. U. "The Nimatron," pp. 330–332.
8. "Pre-Induction Training." pp. 351–352.

Bulletin of the Kansas Association of Teachers of Mathematics

April, 42, Vol. 25, No. 4.

1. Read, Cecil B., "Is Mathematics an Exact Science?" pp. 51–52.
2. Mossman, Thirza, "From Here to the Stars," pp. 52–53.
3. Daugherty, R. D., "Statement and Proof of Napier's Mnemonic Theorem," pp. 53–55.
4. Nyberg, Joseph A., "Saving Time in the Algebra Class," pp. 55–58.
5. Weimar, M. Bird, "New Developments in the Mathematics Curriculum," pp. 59–60.
6. Lewis, Arthur J., "Report on Joint Meetings at Lehigh University," pp. 60–61.
7. James, W. C., "Navy Educational Program," pp. 62–63.
8. Gramlich, Jay I., "Creating New Interest in Arithmetic," pp. 63–64.

National Mathematics Magazine

March 1942, Vol. 16. No. 6.

1. Sanders, S. T., "The Magazine Sponsorship," p. 274.
2. Aude, H. T. R., "Primitive Integral Triangles," p. 280–283.
3. Finkel, Benjamin F., "A History of American Mathematical Journals," (continued) pp. 284–289.
4. Kennedy, E. S., "Interrelations Between Mathematics and Philosophy in the Last Three Centuries," pp. 290–298.
5. Kaltenborn, H. S., "Teaching Centroids and Moments of Inertia Simultaneously," pp. 299–304.

April, 1942, Vol. 16. No. 7.

1. Goldberg, Michael, "Polyhedral Linkages," pp. 323–332.
2. Nichols, Irby C., "Comparison of One, Three, and Five-year Fire Insurance Policies," pp. 338–340.
3. Finkel, Benjamin F., "A History of American Mathematical Journals," pp. 341–344.
4. Reklis, Virginia Modesitt, "The Use of Medians in a Testing Program in Mathematics," pp. 345–349.
5. Stark, Marion E., "Methods of Mathematical Proof for Undergraduates," pp. 350–351.
6. Hart, William L., "Air Corps Recommendations for the Pre-Training of Aviation Cadets," pp. 364–367.

May, 1942, Vol. 16, No. 8.

1. Finkel, Benjamin F., "A History of American Mathematical Journals," pp. 381–391.
2. Schaaf, William L., "The Highest Rung," pp. 392–397.

3. Shapiro, Isidor F., "Adequate Induction by Sampling," pp. 398–399.
4. Gere, Brewster H., "The Construction and Use of a Mathematics Placement Test," pp. 400–406.

School Science and Mathematics

April, 1942, Vol. 42, No. 4.

1. Struyk, Adrian, "Some Additional Notes on a Problem of Steinhaus," pp. 325–330.
2. Jurgensen, R. C., "Discussing Division by 0," pp. 339–341.
3. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 365–368.
4. Read, Cecil B., "A Note on the Use of Directed Lines in Trigonometry," pp. 374–376.

May, 1942, Vol. 42, No. 4.

1. Bowman, Earl C., and Sands, Lester B., "Improving the Education of Mathematics and Science Teachers," pp. 407–412.
2. Colwell, Lewis W., "A Simple Method of Rectifying Small Circles," pp. 419–420.
3. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 463–466.
4. Struyk, Adrian, "The Tangents to a Circle from an Outside Point." pp. 493.

June, 1942, Vol. 42, No. 6.

1. Jamison, G. H., "Will the National Emergency Teach Us Anything in Curriculum Building?" pp. 507–511.
2. LeSourd, Homer W., "Science, Mathematics and the War," pp. 517–519.
3. Pella, Milton O., "Mathematics and Conservation in National Defense," pp. 529–533.
4. Schaaf, William L., "The Dilemma of Vocational Mathematics," pp. 542–549.
5. Nyberg, Joseph A., "Notes from a Mathematics Class Room," pp. 565–568.
6. Greenfield, A., "Algebra and Physics," pp. 571–573.

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New Books Received

Abraham Cohen, ELEMENTS OF CALCULUS, D. C. Heath and Company, 1940 Pp. v+583.

David Raymond Curtiss, and Elton James Moulton, ESSENTIALS OF TRIGONOMETRY WITH APPLICATIONS, D. C. Heath and Company, 1942, Pp. viii+174.+94.

Edward M. Glaser, AN EXPERIMENT IN THE DEVELOPMENT OF CRITICAL THINKING, Bureau of Publication, Teachers College, 1941. Pp. ix+212.

S. L. Green, ALGEBRAIC SOLID GEOMETRY, Cambridge at the University Press, London: Bentley House, 1941, Pp. 132. Price $1.75.

William L. Hart, INTRODUCTION TO COLLEGE ALGEBRA, D. C. Heath and Company, 1937, Pp. v+246.

W. W. Hart, and L. D. Jahn, MATHEMATICS IN ACTION, Book 1, D. C. Heath and Company, 1939, Pp. vii+343.

W. W. Hart, and L. D. Jahn, MATHEMATICS IN ACTION, Book 2, D. C. Heath and Company, 1939, Pp. ix+382.

W. W. Hart, and L. D. Jahn, MATHEMATICS IN ACTION, Book 3, D. C. Heath and Company, 1940, Pp. v+442.

D. E. Hobart, ENGINEERING, D. C. Heath and Company, 1941, Pp. vii+430.

Leo Hudson and E. S. Mills, NATURAL TRIGONOMETRIC FUNCTIONS, Monroe Calculating Machine Company, Inc.

Elisha S. Loomis, THE PYTHAGOREAN PROPOSITION, Edwards Brothers, Inc. Pp. xvi+283. Price $2.00 from Baldwin-Wallace College, Berea, Ohio $2.75 regular price.

James Naidich, MATHEMATICS FOR THE AVIATION TRADES, McGraw-Hill Book Company, Inc. Pp. ix+267. Price $1.80.

W. J. Oliver, P. F. Winters, and J. E. Campbell, A FIRST COURSE IN TRIGONOMETRY, School Aids and Text Book Publishing Company, Pp. 269. Price $1.25.

C. C. Richtmeyer and J. W. Foust, FIRST YEAR COLLEGE MATHEMATICS, F. S. Crofts and Company, Pp. xi+461. Price $3.25.

Raleigh Schorling, R. Rolland Smith, and John R. Clark, SECOND-YEAR ALGEBRA, World Book Company. 1942, Pp. xi+484. $1.68.

Henrietta D. Antoville and Catherine M. Troube (general editor William Jansen) PRACTICAL MATHEMATICS, Book I, Noble and Noble, 1942, Pp. viii+337. $1.04.

Henrietta D. Antoville and Catherine M. Troube (general editor William Jansen) PRACTICAL MATHEMATICS, Book II, Noble and Noble, 1942, Pp. viii+342. $1.04.

Henrietta D. Antoville and Catherine M. Troube (general editor William Jansen) PRACTICAL MATHEMATICS, Book IV, Noble and Noble, 1942, Pp. viii+336. Price $1.04.

E. T. Bell, THE DEVELOPMENT OF MATHEMATICS, McGraw-Hill Book Company, Inc. 1940, Pp. xiii+583. Price $4.50.

C. A. Felker, and H. W. Paine, MILLING-MACHINE-INDEXING, The Bruce Publishing Company, Pp. 88. Price $1.50. 1942.

Charles Hampson Grant, MODEL AIRPLANE DESIGN AND THEORY OF FLIGHT, Jay Publishing Corporation, 1941, Pp. xiii+512. $3.75.

R. V. McGee, MATHEMATICS IN AGRICULTURE, Prentice-Hall, Inc., 1942. Pp. ix+189. $3.00.

Margaret Grace McKim, THE READING OF VERBAL MATERIAL IN NINTH GRADE ALGEBRA, Bureau of Publications, Teachers College, Columbia University, 1941, Pp. viii +133. Price $1.60.

George W. Mixter, PRIMER OF NAVIGATION, D. Van Nostrand Company, Inc. (Fourth Printing) February 1942, Pp. xvii+337. Price $4.00.

Paul R. Rider, PLANE AND SPHERICAL TRIGONOMETRY, The Macmillan Company, 1942, Pp. xiv+275. Price $2.00.

Report of the National Commission on Cooperative Curriculum Planning, THE SUBJECT FIELD IN GENERAL EDUCATION. D. Appleton-Century Company, 1941, Pp. ix+239. Price $1.50.

H. L. Rietz, A. R. Crathorne and L. J. Adams, INTERMEDIATE ALGEBRA, Henry Holt and Company, 1942, Pp. viii+248.

Henry Smithline, and Clyde O. Thompson, A COLLEGE COURSE IN BUSINESS MATHEMATICS, Prentice-Hall, Inc., 1942, Pp. ix +317. Price $1.40.

J. L. Synge and B. A. Griffith, PRINCIPLES OF MECHANICS, McGraw-Hill Book Company, 1942, Pp. xii+514. Price $4.50.

Maxie Nave Woodring, Mervin E. Oakes, and H. Emmett Brown, ENRICHED TEACHING OF SCIENCE IN THE HIGH SCHOOL, Bureau of Publications, Teachers College, Columbia University, 1941, Pp. xii+402. Price $3.25.

Robert C. Yates, THE TRISECTION PROBLEM, The Franklin Press, 1942, Pp. 68.

NEWS NOTES

The last meeting of the Men's Mathematics Club of Chicago for the year 1941–42 was held on April 17th. Hans Gutekunst of the Batavia High School spoke on the topic "Streamlining Solid Geometry for Defense" and Edwin W. Schreiber of the Macomb State Teachers College discussed "A New Visual Aid for Teaching Mathematics."

At this meeting the following officers were elected for the year 1942–43:

President: S. F. Bibb, Illinois Institute of Technology

Secretary-Treasurer: M. D. Oestricher, Francis W. Parker High School

Recording Secretary: Glenn F. Hewitt, Von Steuben High School.

The Sixth Annual Meeting of The Nebraska Section of the National Council of Teachers of Mathematics was held at the University of Omaha on Saturday, May 2, 1942.

Executive Board

President—Mildred I. Clark, Wayne

Vice-president—Milton Beckmann, Wayne

Secretary—Ruth A. Thompson, Minden

Treasurer—Roxie McGrew, Auburn

Bulletin Committee—R. Perry Anderson, Genoa, Mary Heartwell, Alliance

N. C. T. M. Representative—Dr. A. R. Congdon, University of Nebraska.

Program

Theme: *Defense Needs Met Mathematically*

Part I. 9:00 A.M.

Business Meeting and Election of Officers.

"What Can High School Teachers of Mathematics Do to Contribute Toward Defense Training." W. T. Stratton, Head of Mathematics Department, Kansas State College, Manhattan, Kansas.

Part II. Joint Meeting With the Mathematical Association of America. 10:15 A.M.

"Mechanical Apparatus and Elementary Nomograms for the Construction and Illustration of Harmonic Curves and Stereographs Associated with them." L. E. Smith and A. B. Trussell, both of Omaha South High School.

"Demonstrations by Means of Oscillograms of Lissajou and Other Curves of Value in Certain Ultrafrequency Work Used by the Signal Corps." T. T. Smith, University of Nebraska.

"Precision Construction of the Curves Demonstrated Above by Means of the Harmonic Analyzer." M. G. Gaba, University of Nebraska.

Part III. 12:45 P.M.

Joint Luncheon with members of the Mathematical Association of America.

Luncheon Speakers:

Highlights of N. C. T. M. Convention of San Francisco—Dr. A. R. Congdon, University of Nebraska

Mathematics in the War Program—W. C. Brenke, University of Nebraska.

The Nassau County (N. Y.) Mathematics Teachers' Association held five meetings during the past year (1941–42)—four business and one social. The officers were:

President: Alice M. Reeve, Rockville Center

Vice-President: Ezra Reed, Baldwin

Secretary-Treasurer: Zaven Mahdesian, Great Neck.

The Winter Meeting of the Portland Council of Teachers of Mathematics was held Dec. 6, 1941, at the Heathman Hotel, Portland, Oregon.

At the morning session Lieutenant H. R. Johnson, Classification Officer at the Portland Columbia Airbase explained the use made of tests in placing men in positions for which they are best qualified.

Miss Lesta Hoel, Supervisor of Mathematics in the Portland schools gave a paper on "Fractions."

The officers elected for 1942 are:

President: Mr. Robert Main, Washington High School. (Home) 3736 N.E. 80th St., Portland, Oregon.

Vice-President: Miss Katherine Piggott, Jefferson H. S. (Home) 1409 N.E. Hancock St., Portland, Oregon.

Secretary-Treasurer: Mr. A. B. Carter, Grant H. S. (Home) 6635 S.E. Yambill St., Portland, Oregon.

Corresponding Secretary: Miss Agnes Beach, Washington H. S. (Home) 2328 S.W. 18th St., Portland, Oregon.

SPRING MEETING OF THE MATHEMATICS SECTION COLORADO EDUCATION ASSOCIATION

The spring meeting of the Mathematics Section of the Eastern Division of the Colorado Education Association held a luncheon meeting in Denver, April 11.

The theme of the meeting was "Mathematics Forges Ahead."

9:30—Exhibit at East High School by the Euclidian Club, H. W. Charlesworth, Sponsor.

"The House of Modern Magic," Mr. G. E.

Lindeman of the Public Service Company of Colorado.

12:30 Luncheon, Daniels and Fishers Tea Room.

Discussion Topics.

"Mathematics as a Contributing Subject," Amanda A. Lindsey, Byers Junior High School. Denver.

"Is Mathematics Taught Incidentally?" Isabel Marsh Lody, Gove Junior High School, Denver.

"Mathematics at Work in a Modern Setting," Ruth Irene Hoffman, Skinner Junior High School, Denver.

"Mathematics and Defense," Lieutenant Ross, Lowry Field, Denver.

President: Wendell Wolff

Secretary: Valworth Plumb

Annual Mathematics Exhibit Put on by East High Euclidians

Four Day Affair Centers Around Patriotic Theme; Projects Sponsored by Departments and Clubs.

Red, white and blue paper streamers, blue drapes, red, white and blue stellated icosahedrons decorated the boys' "gym" at East High School during the Fourth Annual Mathematics Exhibit of the Euclidian Club sponsored by H. W. Charlesworth.

From all corners and sides of the balcony the twisted, colored streamers ran to the center of the gym ceiling. There were numerous icosahedrons, much smaller than the center one, placed around the center attraction. Along the sides of the balcony were banners and shields which furthered the patriotic theme.

Various school departments and clubs sponsored individual mathematical projects. Among those participating were the physics department, the Bible club, the Astronomy club, the Pre-medics club and the mathematics department.

According to Herbert Hubbard, club president, over 400 East High students participated in making the exhibit. Numerous local merchants also assisted in making the exhibit possible.

The exhibit was given for visiting teachers and students from schools outside of Denver on Saturday April 11. It was repeated again June 30 to July 1, when the National Council of Teachers of Mathematics met in Denver at the time of the N.E.A. Convention.

The Rocky Mountain Section of the Mathematical Association of America held its 26th Annual Meeting at the Colorado School of Mines in Golden, Colorado on April 17 and 18, 1942.

Chairman: J. C. Fitterer

Vice Chairman: A. E. Mallory

Secretary-Treasurer: Arthur J. Lewis

Program

Friday, April Seventeenth

3:00 P.M.—Guggenheim Hall, Room 342

An Improved Cosine-Law Slide Rule (15 minutes). *Professor Ivan L. Hebel,* Colorado School of Mines.

On Determining the "Best" Critical Region for Testing the Null Hypothesis When the Parent Populations Follow the Poisson Law (30 minutes). *Mr. H. T. Guard,* Colorado State College.

The Use of Cauchy's Integral Formula in Evaluating Certain Improper Integrals (30 minutes). *Professor Arthur J. Lewis,* University of Denver.

A Study of Roulettes With the Aid of the Cathode-Ray Oscillograph—A Condensation (30 minutes). *Professor Myron G. Pawley,* Colorado School of Mines.

On Some Maximum and Minimum Problems in Geometry (20 minutes). *Mr. George E. Uhrich,* University of Colorado.

6:30 Dinner

Greetings: *President M. F. Coolbaugh,* Colorado School of Mines.

Response: *Professor A. E. Mallory,* Colorado State College of Education.

8:00 P.M.

Guggenheim Hall, Room 342

Mathematical Approach to the Fundamentals of Hydraulics. *Mr. C. P. Vetter, Senior Engineer,* Bureau of Reclamation.

Remarks on Repeating Decimal Fractions, *Professor Emeritus Ira M. DeLong,* Boulder, Colorado. (Read by Professor Aubrey J. Kempner.)

Periodic Decimal Fractions, Primitive Roots, and Quadratic Residues. *Professor Aubrey J. Kempner,* University of Colorado.

Saturday, April Eighteenth

9:00 A.M.—Guggenheim Hall, Room 342

Business.

Joint Session with the Mathematics Section of the Eastern Division of the Colorado Educational Association. *H. W. Charlesworth,* Denver Public Schools, Presiding.

A Report of Instruction and Learning Activities as Observed in Geometry Class Rooms. *Professor A. E. Mallory,* Colorado State College of Education.

The Handmaiden Spurned. *Professor O. H. Rechard.* University of Wyoming.

On a Simple Proof of the Addition Theorem in Trigonometry. *Mr. George E. Uhrich,* University of Colorado.

The Use of Mathematics in Industry. *Mr. Louis A. McElroy,* Coordinator of Trade and Apprentice Training, Denver Public Schools.

Construction by Means of a Marked Ruler and Other Instruments. *Professor Claribel Kendall,* University of Colorado.

The Mathematics Section of the Eastern Division of the Colorado Education Association accepted the following resolutions at its Spring Meeting in Denver on April 11.

These resolutions will be presented to superintendents and principals in this section of Colorado.

The resolutions were accepted by the Rocky Mountain Section of the Mathematical Association of America as the minimum requirements in the hope that higher standards might be attained in the future.

Report of Resolutions Committee
Mathematical Association of America
Rocky Mountain Section

1. Because modern conditions show ever increasing needs of a knowledge of mathematics and demand great numbers of people with training in mathematics,

2. Because the advancement of science depends to a large degree on mathematics,

3. Because many students desire to become engineers and discover they do not have the prerequisite knowledge of mathematics when they enter college,

4. Because many students acquire a distaste for mathematics and fail to grasp the subject matter of mathematics as a result of studying under poorly prepared teachers,

5. And because the very nature of mathematics is such that it cannot be learned incidentally;

We, therefore, recommend:

1. That a minimum of three years of mathematics be available for capable students in the high schools beginning with the ninth grade;

2. That dompetence in arithmetic be expected of all graduates; (All students throughout the senior high school should be checked through tests as to their ability in arithmetic concepts, reasoning and fundamental skills. Those found wanting should be required to remedy this situation.)

3. That only qualified teachers of mathematics be employed to teach mathematics. (Only those teachers who have been trained in the teaching of mathematics and who are vitally interested in this subject can obtain satisfactory results.)

Resolutions Committee
CHARLES HUTCHINSON
HARL DOUGLASS
ARTHUR J. LEWIS
H. W. CHARLESWORTH
ALFHILD ALENIUS

The Spring meeting of the Connecticut Valley Section of the Association of The Teachers of Mathematics in New England was held on April 18, 1942 at the Paul Mellon Science Hall of the Choate School in Wallingford, Conn.

PROGRAM

Morning Session

10:30 Registration.

11:00 Address of Welcome. *Dr. George St. John*, Headmaster of The Choate School.

11:15 Round Table Discussion. Subject: Is Education in the United States a Failure?

Panel Chairman: Miss Dorthy S. Wheeler, 2nd Vice-President of the National Council of Teachers of Mathematics.

Panel Speakers: *Dr. John S. Brubacher*, Department of Education, Yale University. *Mr. Clarence E. Hale*, Investment Banker, New York City. *Dr. Herbert B. Welte*, President, Connecticut State Teachers College. *Mr. Courtenay Hemenway*, The Choate School.

12:15 Question Period.

1:00 Luncheon at the School.

Afternoon Session

2:00 Annual Business Meeting and Election of Officers.

OFFICERS

of the Connecticut Valley Section

William G. Shute, *President*, The Choate School

Maytscherl Walsh, *Vice-President*, Bulkeley High School, Hartford

Julia Wells Bower, *Secretary*, Connecticut College

Ethelyn M. Percival, *Treasurer*, Westfield High School

Marie Litzinger, *Director*, Mount Holyoke College

C. H. W. Sedgewick, *Director*, University of Connecticut

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M. M. S. Moriarty	'22
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